

Math 255 - Spring 2022
The division algorithm

This homework contains proofs that use the Division Algorithm as a crucial step. This homework is worth 10 points.

1. Use the Division Algorithm to show that the square of any integer is of the form $3k$ or $3k + 1$.
2. Use the Division Algorithm to show that the square of any integer is of the form $4k$ or $4k + 1$.
3. For $n \geq 1$, prove that $\frac{n(n+1)(2n+1)}{6}$ is an integer.
(Hint: By the Division Algorithm, $n = 6k, 6k + 1, \dots$ or $6k + 5$. Prove the result in each case separately.)