Math 395 - Spring 2020
Homework 6

This homework is due on Monday, February 24.

All of these problems must be typed up.

1. Let R be a Principal Ideal Domain, let M be an R-module, and let p be a nonzero prime in R. Define

$$M_p = \{m \in M \mid p^a m = 0 \text{ for some } a \in \mathbb{Z}^+\},$$

called the p-primary component of M.

   (a) Prove that M_p is an R-submodule of M.
   (b) Prove that $(M/M_p)_p = 0$, i.e., the p-primary component of M/M_p is zero.
   (c) Prove that if q is a nonzero prime in R different from p, then $M_p \cap M_q = 0$.

2. Let R be a Principal Ideal Domain with field of fractions F and assume $R \neq F$. As usual we may view F as a module over its subring R.

   (a) Prove that every finitely generated R-submodule of F is a cyclic R-module.
   (b) Deduce from (a) that F cannot be a finitely generated R-module.

   (You may quote results about modules over PID.)