MATH 295 QUIZ 2 JANUARY 22, 2020

Name:

Problem 1: *Let B be a nonempty set, and suppose that there is a positive integer n and an injective map*

$$f\colon B \to \{1, 2, \ldots, n\}.$$

Prove that B is finite.

Solution: Let A be the image or range of f (i.e.

$$A = \{a \in \{1, 2, \ldots, n\} \mid \exists b \in B \text{ with } f(b) = a\}).$$

Then A is a subset of $\{1, 2, \ldots, n\}$ and therefore finite (a subset of a finite set is finite by Corollary 6.6 of Chapter 1). Then $f\colon B \to A$ is injective since f is injective, and surjective since A is the image of f, and therefore there is a bijection from B to a finite set and B is finite.