Name:

Problem 1: *Let X be a topological space, A be a subspace of X, and $x_n \in A$ for $n = 1, 2, \ldots$ Suppose that the sequence $\{x_n\}$ converges to $x \in X$. Show that $x \in \overline{A}$.*

For partial credit, please give the definition of "$\{x_n\}$ converges to x" for topological spaces.

Solution: The definition of "$\{x_n\}$ converges to x" is the following: For every open neighborhood $U \subset X$ of x, all but finitely many elements of the set $\{x_n\}_{n=1}^{\infty}$ belong to U.

Now let x be the limit of a sequence $\{x_n\}$ with $x_n \in A$ for all n. Let U be an open neighborhood of x. Since x_n converges to x, certainly U contains at least one element of the sequence x_n, and therefore an element of A. It follows that for all U open containing x, $U \cap A \neq \varnothing$, and $x \in \overline{A}$.