Math 295 - Spring 2020
Homework 11

This homework is due on Wednesday, April 1. All problems are adapted from Munkres's *Topology*.

1. Let $\mathcal{T}, \mathcal{T}'$ be two topologies on the set X. If $\mathcal{T}' \supset \mathcal{T}$, what does connectedness of X in the $\mathcal{T}$ topology imply about connectedness in the $\mathcal{T}'$ topology? Conversely, what does connectedness of X in the $\mathcal{T}'$ topology imply about connectedness in the $\mathcal{T}$ topology?

2. Let $\{A_n\}$ be a sequence of connected subspaces of X, such that $A_n \cap A_{n+1} \neq \varnothing$ for all n. Show that $\bigcup A_n$ is connected.

3. A space is *totally disconnected* if its only connected subspaces are one-point sets. Show that if X has the discrete topology, then X is totally disconnected. Does the converse hold?

Extra problem for graduate credit:

1. Let $Y \subset X$ with X and Y connected, Y a subspace of X. Show that if A and B form a separation of $X - Y$, then $Y \cup A$ and $Y \cup B$ are connected.