Math 295: Spring 2020
Exam 2

**NAME:**

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

If you cannot print the exam, please indicate clearly which problem you are solving on your turned in work. Please only solve one problem per page.

You have until 12:10pm to upload your exam to BlackBoard. If you have technical difficulties that make you miss this deadline, please contact me directly.

| Problem | Value | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 16 | |
| 3 | 24 | |
| GC | 5 | |
| TOTAL | 50 (or 55) | |

**Problem 1 : (10 points) Please read this entire question carefully.**

Note that failure to abide by the rules set below will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access the internet during the exam for any reason, except to download the exam and then later scan and upload your work.
- **The work you present must be your own.**
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, with which you should familiarize yourself if you haven't already.

For ten points, please copy the following statement on your turned in work: "I solemnly swear that the work presented therein is my own, and that I did not access any notes, websites, or other person to aid me in answering these questions."

**Problem 2 : (16 points)**

a) (8 points) Let $X$ be a topological space. Give the definition of what it means for $X$ to be *connected*.

b) (8 points) Let $X$ be a topological space. For $J$ an arbitrary indexing set and $\alpha \in J$, suppose that $A_\alpha \subset X$ is a connected subspace. Suppose also that $A \subset X$ is a connected subspace of $X$, such that for each $\alpha \in J$, $A_\alpha \cap A \neq \varnothing$. Show that

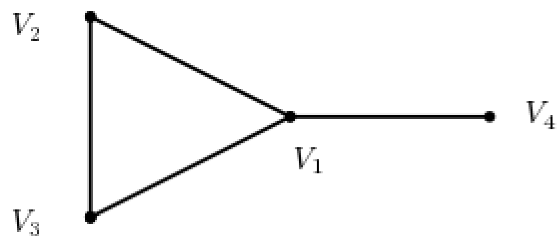
$$A \bigcup \left( \bigcup_{\alpha \in J} A_\alpha \right)$$

is connected.

**Problem 3 : (24 points)** Let $X = \{V_1, V_2, \ldots, V_n\}$ be the set of vertices of a graph, and let $d\colon X \times X \to \mathbb{R}$ be given by the formula:

$$d(V_i, V_j) = \text{length of the shortest path from } V_i \text{ to } V_j \text{ in the graph.}$$

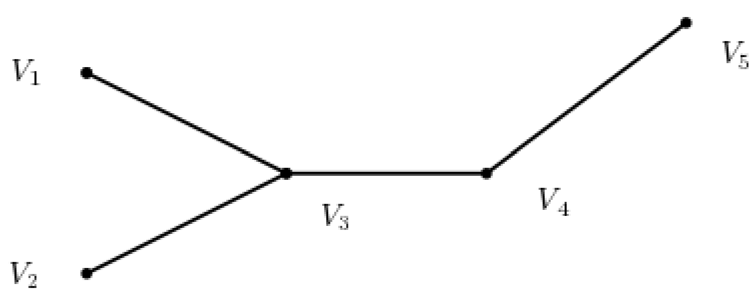
For example, consider the following graph with set of vertices $X = \{V_1, V_2, V_3, V_4\}$.




Then we have

$$\begin{aligned}
&d(V_1,V_1)=0, \quad d(V_1,V_2)=1, \quad d(V_1,V_3)=1, \quad d(V_1,V_4)=1,\\
&d(V_2,V_1)=1, \quad d(V_2,V_2)=0, \quad d(V_2,V_3)=1, \quad d(V_2,V_4)=2,\\
&d(V_3,V_1)=1, \quad d(V_3,V_2)=1, \quad d(V_3,V_3)=0, \quad d(V_3,V_4)=2,\\
&d(V_4,V_1)=1, \quad d(V_4,V_2)=2, \quad d(V_4,V_3)=2, \quad d(V_4,V_4)=0.
\end{aligned}$$

a) (8 points) Let $X = \{V_1, V_2, V_3, V_4, V_5\}$ be the set of vertices of the graph below:




Compute the following:

i. $d(V_1, V_2)$

iii. $d(V_1, V_5)$

ii. $d(V_2, V_1)$

iv. $d(V_2, V_4)$

b) (8 points) Give the definition of a *metric* on a set $X$.

c) (8 points) Explain (using English words) why $d$ is a metric.

**Extra problem for graduate credit:**

**Problem 4 : (5 points)** Recall that if $X$ is a set, the *finite complement topology* on $X$ consists of the collection of subsets of $X$ whose complement in $X$ is finite.

Let $X$ be an infinite set with the finite complement topology. Show that $X$ is connected.