

Name:

Problem 1: *The Diophantine equation*

$$6x + 14y = 152$$

has solutions parametrized by the equations

$$\begin{aligned}x &= 2 + 7t, \\y &= 10 - 3t,\end{aligned}$$

where t is an integer.

*Give the set of solutions where x and y are **positive** integers.*

You may give your answer by either explicitly enumerating either the elements of the set or the values of t giving those elements.

Solution: If $x > 0$, then we have

$$\begin{aligned}2 + 7t &> 0 \\7t &> -2 \\t &> \frac{-2}{7}.\end{aligned}$$

Since t is an integer, this means that $t \geq 0$.

If $y > 0$, then we have

$$\begin{aligned}10 - 3t &> 0 \\10 &> 3t \\ \frac{10}{3} &> t.\end{aligned}$$

Since t is an integer, this means that $t \leq 3$.

Therefore the only values of t that give both $x > 0$ and $y > 0$ are $t = 0, 1, 2$, and 3 . Explicitly, the integer solutions are the pairs (x, y) given by

$$(2, 10), \quad (9, 7), \quad (16, 4), \quad \text{and} \quad (23, 1).$$