Name:

Problem 1: *Give one pair of integers x and y such that*

$$314x + 159y = 1.$$

(Note that $(314, 159) = 1$, so this is possible.)
To receive credit for this question, you must show your work.

Solution: We will do the Euclidean Algorithm and then back-substitution.
We have

$$\begin{aligned} 314 &= 159 \cdot 1 + 155 \\ 159 &= 155 \cdot 1 + 4 \\ 155 &= 4 \cdot 38 + 3 \\ 4 &= 3 \cdot 1 + 1 \\ 3 &= 1 \cdot 3 + 0. \end{aligned}$$

Then we solve for all of the remainders:

$$\begin{aligned} 155 &= 314 - 159 \\ 4 &= 159 - 155 \\ 3 &= 155 - 4 \cdot 38 \\ 1 &= 4 - 3. \end{aligned}$$

Finally, we back-substitute each remainder into the next equation, starting at the bottom. We start with putting the second equation from the bottom into the bottom equation:

$$\begin{aligned} 1 &= 4 - 3 \\ &= 4 - (155 - 4 \cdot 38) \\ &= 4 - 155 + 38 \cdot 4 \\ &= 39 \cdot 4 - 155. \end{aligned}$$

Then we put the third equation from the bottom into our equation:

$$\begin{aligned} 1 &= 39 \cdot 4 - 155 \\ &= 39 \cdot (159 - 155) - 155 \\ &= 39 \cdot 159 - 39 \cdot 155 - 155 \\ &= 39 \cdot 159 - 40 \cdot 155. \end{aligned}$$

Finally we put the top equation into our equation:

$$\begin{aligned} 1 &= 39 \cdot 159 - 40 \cdot (314 - 159) \\ &= 39 \cdot 159 - 40 \cdot 314 + 40 \cdot 159 \\ &= 79 \cdot 159 - 40 \cdot 314. \end{aligned}$$

We get the solution $x = -40$ and $y = 79$.