

Name:

Problem 1: *In this problem we will explore Lemma 1 of Section 6. Let $p = 11$ and $a = 4$.*

*For each of the following integers, compute their **least residue** modulo 11:*

$$4, 2 \cdot 4, 3 \cdot 4, 4 \cdot 4, 5 \cdot 4, 6 \cdot 4, 7 \cdot 4, 8 \cdot 4, 9 \cdot 4, 10 \cdot 4.$$

*What do you notice? Write **one** sentence.*

Solution: The integers are

$$4, 8, 12, 16, 20, 24, 28, 32, 36, 40,$$

and their least residues modulo 11 are, in order:

$$4, 8, 1, 5, 9, 2, 6, 10, 3, 7.$$

We notice that this is the complete list of least residues modulo 11, except for 0. Alternatively, we might notice that all of the least residues are different.