Name:

Problem 1: *Let x be an integer such that*

$$x \equiv 1 \pmod{3}.$$

Please list all possible congruence classes that x can belong to modulo 9.

Solution: If $x \equiv 1 \pmod 3$, then $x = 1 + 3q$ for some integer q, by Theorem 1 of Section 4.

We note that q itself can be of one of three forms: $q = 3m$ for m an integer, $q = 1 + 3m$ for m an integer and $q = 2 + 3m$ for m an integer, by the Division Algorithm.

If $q = 3m$, then

$$x = 1 + 3(3m) = 1 + 9m,$$

and therefore $x \equiv 1 \pmod 9$.

If $q = 1 + 3m$, then

$$x = 1 + 3(1 + 3m) = 1 + 3 + 9m = 4 + 9m,$$

and therefore $x \equiv 4 \pmod 9$.

Finally, if $q = 2 + 3m$, then

$$x = 1 + 3(2 + 3m) = 1 + 6 + 9m = 7 + 9m,$$

and therefore $x \equiv 7 \pmod 9$.

Therefore, if $x \equiv 1 \pmod 3$, then $x \equiv 1, 4$ or $7 \pmod 9$.