Name:

Problem 1:   *How many solutions does the equation*

$$12x \equiv 8 \pmod{16}$$

have?
For full credit you must justify your answer.

Solution: This is an equation of the form $ax \equiv \pmod{m}$, with $a = 12$, $b = 8$ and $m = 16$. We note that $(12, 16) = 4$ and $4|8$. Therefore by Lemma 3, this equation has 4 solutions.
(Note that if $(12, 16)$ did not divide b, then there would be no solution.)

We will even solve these equations: First we divide all the way through by 4 to get the equation

$$3x \equiv 2 \pmod{4}.$$

Now 3 is a unit modulo 4, with inverse 3 (because $3 \cdot 3 = 9 \equiv 1 \pmod 4$). Therefore multiplying both sides by 3, we get

$$x \equiv 6 \equiv 2 \pmod{4}.$$

Now we lift $x \equiv 2 \pmod 4$ to all of its solutions modulo 16. They are

$$x = 2 + 4q,$$

for $q = 0, 1, 2, 3$. This gives

$$x \equiv 2, 6, 10, 14 \pmod{16}.$$

Note that if we continue with $q = 4, 5, 6, \ldots$, all we do is get the same solutions again, so there are no more.