Name:

**Problem 1:** *Please compute* $8^{-1}$ *modulo* 59.

*In other words, the class* $[8]_{59}$ *is a unit. Please compute its inverse* $u$*, which is the class* $[u]_{59}$ *such that*

$$[8]_{59} \times [u]_{59} = [1]_{59}.$$

**Solution:** We can solve this problem by inspection, by trying to find another integer $u$ such that $8u \equiv 1 \pmod{59}$. Sometimes that is easy and quick, but here I don't see anything immediately, so I will do the process.
We begin with the Euclidean algorithm:

$$\begin{aligned} 59 &= 8 \cdot 7 + 3 \\ 8 &= 3 \cdot 2 + 2 \\ 3 &= 2 + 1. \end{aligned}$$

Then we back-solve to solve the equation $8x + 59y = 1$. The solution $x$ is the inverse we seek:

$$\begin{aligned} 1 &= 3 - 2 \\ &= 3 - (8 - 2 \cdot 3) \\ &= 3 - 8 + 2 \cdot 3 \\ &= 3 \cdot 3 - 8 \\ &= 3(59 - 7 \cdot 8) - 8 \\ &= 3 \cdot 59 - 21 \cdot 8 - 8 \\ &= 3 \cdot 59 - 22 \cdot 8. \end{aligned}$$

Therefore, from the equation $(-22) \cdot 8 - 1 = (-3) \cdot 59$, we get that 59 divides $(-22) \cdot 8 - 1$ or $(-22) \cdot 8 \equiv 1 \pmod{59}$.
Therefore $8^{-1} \equiv -22 \pmod{59}$, or if we prefer a least residue, $8^{-1} \equiv 37 \pmod{59}$.