Math 255 - Spring 2018
Homework 6

This homework is due on Monday, February 26.

1. Prove Lemma 6 from the book. In other words, show that if $p$ is a prime, $a_1, a_2, \ldots, a_m$ are integers and $p$ divides the product $a_1 a_2 \cdots a_m$, then $p$ must divide $a_i$ for some $1 \leq i \leq m$.

2. Use induction to prove Lemma 2 from the book. In other words, use induction on $n$ to show that if $n > 1$ is an integer, then $n$ is a product of primes.

   Note: To receive credit for this problem you **must** use induction on $n$. You may need to use strong induction; that is okay.

3. In this problem we will use the Sieve of Eratosthenes to find all primes that are less than 200.

   (a) According to the book on page 14, to find all primes less than $N^2$, it suffices to eliminate the multiples of all of the primes that are less than or equal to $N$. Let $N$ be the **integer** such that, to find all primes that are less than 200, we must eliminate all of the multiples of all of the primes that are less than or equal to $N$. What is $N$ here?

   (b) Use the Sieve of Eratosthenes to find all primes that are less than 200. You may use the grid posted online, and you may give your answer to this question as the grid with all primes circled.

4. Let $n$ be composite, and let $p$ be the smallest prime factor of $n$. Prove that if $p > n^{1/3}$, then $\frac{n}{p}$ is a prime number.

Extra problem for graduate credit:

5. Show that if $n$ is composite, then $2^n - 1$ is also composite. Is the converse true? (In other words, is it true that if $p$ is prime, then $2^p - 1$ is also prime?)