

Math 255 - Spring 2018  
Homework 2

This homework is due on Monday, January 29.

1. Let  $a$  and  $b$  be integers, not both of which are zero. Prove that  $(a, b) = ((a, b), b)$ .
2. Let  $n$  and  $k$  be integers, not both of which are zero. Prove that  $(k, n + k) = 1$  if and only if  $(k, n) = 1$ .
3. Prove that if  $a|b$  and  $a > 0$  then  $(a, b) = a$ .

Extra problem for graduate credit:

4. Let  $a$  and  $b$  be integers, not both of which are zero. Show that if  $d = (a, b)$  and  $c$  is a common divisor of  $a$  and  $b$ , then  $c|d$ . (Hint: You will need to use Theorem 4.)

Note that this is enough to show that when  $a$  and  $b$  are not zero, both definitions of the greatest common divisor given in class are equivalent.