Math 255 - Spring 2018
Homework 10

This homework is due on Monday, April 9.

1. What is the remainder when 2018^{2018} is divided by 26?
2. What is the last digit of 2018^{2018}?
3. Let $p \geq 5$ be a prime. Show that $2(p-3)! + 1 \equiv 0 \pmod{p}$.
4. Show that $18! \equiv -1 \pmod{437}$.
5. For $n \geq 1$, $n \in \mathbb{Z}$, let
$$f(n) = \sum_{d|n} (d+1).$$
   (a) Give a closed formula for $f(n)$, similar to those we gave in class for $\sigma(n)$ and $d(n)$.
   (b) Is f a multiplicative function? Prove or disprove.

Extra problem for graduate credit:

6. We have that
$$\begin{aligned} 6! &\equiv -1 \pmod{7} \\ 5!1! &\equiv 1 \pmod{7} \\ 4!2! &\equiv -1 \pmod{7} \\ 3!3! &\equiv 1 \pmod{7}. \end{aligned}$$
   (a) Perform the same sort of calculations modulo 11.
   (b) Guess a theorem from the data given to you and your data from part (a), and prove it.