

Math 255: Spring 2016
Midterm 2

NAME:

SOLUTIONS

Time: 50 minutes

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: _____

Problem	Value	Score
1	4	
2	5	
3	5	
4	8	
5	8	
6	8	
7	12	
TOTAL	50	

Problem 1 : (4 points) What is the order of 4 modulo 17?

$\phi(17)=16$ so the order of 4 divides 16

$$4^2 = 16 \equiv -1 \pmod{17}$$

$$4^4 \equiv (-1)^2 \equiv 1 \pmod{17}$$

4 has order 4 modulo 17.

Problem 2 : (5 points) What is 22^{-1} modulo 47?

Euclidean algorithm:

$$47 = 2 \cdot 22 + 3$$

$$3 = 47 - 2 \cdot 22$$

$$22 = 7 \cdot 3 + 1$$

$$\rightarrow 1 = 22 - 7 \cdot 3$$

$$1 = 22 - 7(47 - 2 \cdot 22)$$

$$= 22 - 7 \cdot 47 + 14 \cdot 22$$

$$= 15 \cdot 22 - 7 \cdot 47$$

So $22^{-1} \equiv 15 \pmod{47}$