Math 255 - Spring 2017
Answers to selected suggested problems
Problems up to March 1 (Exam 1)

Please note: If there are any typos, please post about them on Piazza. The latest corrections to the solutions will be available there.

Section 1.1

5. (b) See Quiz 1 solutions

Section 2.2

2. See Quiz 3 solutions

Section 2.3

14. (b) We have that
$$-7(5a + 2) + 5(7a + 3) = 1,$$
and therefore the equation $(5a + 2)x + (7a + 3)y = 1$ has a solution for every integer a. (This solution is $x = -7, y = 5$.) By Theorem 2.4, this implies that $\gcd(5a + 2, 7a + 3) = 1$.

Section 2.4

1. $\gcd(143, 227) = 1$, $\gcd(306, 657) = 9$, and $\gcd(272, 1479) = 17$
2. (a) $\gcd(56, 72) = 8$ and a solution is $x_0 = 4$, $y_0 = -3$
   (b) $\gcd(24, 138) = 6$ and a solution is $x_0 = 6$, $y_0 = -1$
   (c) $\gcd(119, 272) = 17$ and a solution is $x_0 = 7$, $y_0 = -3$
   (d) $\gcd(1769, 2378) = 29$ and a solution is $x_0 = 39$, $y_0 = -29$

Section 2.5

1. Equation (a) cannot be solved because 3 does not divide 22, equation (b) can be solved because $\gcd(33, 14) = 1$, equation (c) cannot be solved because 7 does not divide 93.
2. (a) $x = 20 + 9t, y = -15 - 7t, t \in \mathbb{Z}$
   (b) $x = 18 + 23t, y = -3 - 4t, t \in \mathbb{Z}$
   (c) $x = 176 + 35t, y = -1111 - 221t, t \in \mathbb{Z}$
3. (a) The only solution in the positive integers is $x = 1, y = 6$
   (c) There are no positive integer solutions

Section 3.1

5. (a) See Quiz 8 solutions

Section 3.2

1. 701 and 1009 are both prime

2. The primes between 100 and 200 are: $101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197$, and 199

Section 4.2

11. We have

$$\begin{aligned}
2^2 &= 4 \pmod{11} \\
2^3 &= 8 \pmod{11} \\
2^4 &= 16 \equiv 5 \pmod{11} \\
2^5 &= 2^4 \cdot 2 \equiv 5 \cdot 2 = 10 \pmod{11} \\
2^6 &= 2^5 \cdot 2 \equiv 2 \cdot 10 = 20 \equiv 9 \pmod{11} \\
2^7 &= 2^6 \cdot 2 \equiv 9 \cdot 2 = 18 \equiv 7 \pmod{11} \\
2^8 &= 2^7 \cdot 2 \equiv 7 \cdot 2 = 14 \equiv 3 \pmod{11} \\
2^9 &= 2^8 \cdot 2 \equiv 3 \cdot 2 = 6 \pmod{11}.
\end{aligned}$$

By inspection this covers all of the congruence classes modulo 11 between 3 and 10, inclusively. Therefore if we add in $0, 1$ and 2, this covers all of the congruence classes modulo 11 and the set is a complete set of residues modulo 11.

However, we have

$$\begin{aligned}
1^2 &= 1 \pmod{11} \\
2^2 &= 4 \pmod{11} \\
3^2 &= 9 \pmod{11} \\
4^2 &= 16 \equiv 5 \pmod{11} \\
5^2 &= 25 \equiv 3 \pmod{11} \\
6^2 &\equiv (-5)^2 = 5^2 \equiv 3 \pmod{11} \\
7^2 &\equiv (-4)^2 \equiv 5 \pmod{11} \\
8^2 &\equiv (-3)^2 = 9 \pmod{11} \\
9^2 &\equiv (-2)^2 = 4 \pmod{11} \\
10^2 &\equiv (-1)^2 = 1 \pmod{11}
\end{aligned}$$

We see that we have not obtained the congruence classes $2, 6, 7$, and 8 modulo 11. The squares do not form a complete set of residues modulo 11.

Section 4.4

1. (a) $x \equiv 18 \pmod{29}$
   (b) $x \equiv 16 \pmod{26}$
   (c) $x \equiv 6, 13, 20 \pmod{21}$
   (d) No solution because 6 does not divide 8
   (e) $x \equiv 45, 94 \pmod{98}$
   (f) $x \equiv 16, 59, 102, 145, 188, 231, 274 \pmod{301}$