Name:

Problem 1: *Consider the Chinese Remainder Theorem.*

Can the Chinese Remainder Theorem be applied to determine if the system of linear congruences

$$\begin{aligned} x &\equiv 3 \pmod 4 \\ x &\equiv 5 \pmod 6 \\ x &\equiv 1 \pmod 7 \end{aligned}$$

has a simultaneous solution?

To support your answer, either go through each hypothesis of the theorem and check that it is satisfied; or give (at least) one hypothesis of the theorem that is not satisfied.

Solution: The Chinese Remainder Theorem **cannot** be applied to determine if this system has a simultaneous solution. This is because here $n_1 = 4$, $n_2 = 6$ and $n_3 = 7$ and $\gcd(n_1, n_2) = \gcd(4, 6) = 2 > 1$. Therefore the hypothesis is not satisfied for all of the n_is and the theorem does not apply.