Math 255 Quiz 8 February 8 2017

Name:

Problem 1: *Let p be a prime. Prove that if p divides a^n for $a, n \in \mathbb{Z}$ and $n > 0$, then p^n divides a^n.*

Solution: We apply Corollary 1 proved in class: If p is a prime and p divides a product of n integers, then p divides one of the integers in the product. Here the product of n integers is

$$a^n = a \cdot a \cdot a \dots a,$$

so we can say that $p|a^n$ implies $p|a$.

Now if $p|a$, this means that we can write $a = kp$ for $k \in \mathbb{Z}$. Therefore we have

$$a^n = (kp)^n = k^n p^n.$$

Since $k^n \in \mathbb{Z}$, it follows that p^n divides a^n.