Name:

Problem 1: *Give one integer solution for the equation*

$$6 = 24x + 138y.$$

Note that $\gcd(24, 138) = 6$.

Solution: The first step is to do the Euclidean algorithm:

$$138 = 5 \times 24 + 18$$
$$24 = 1 \times 18 + 6$$
$$18 = 3 \times 6.$$

(We see that indeed the gcd is 6.)

The second step is to solve for the remainder in each step but the last:

$$18 = 138 - 5 \times 24$$
$$6 = 24 - 18.$$

Now we back-substitute upwards:

$$6 = 24 - 18$$
$$= 24 - (138 - 5 \times 24)$$
$$= 24 - 138 + 5 \times 24$$
$$= 6 \times 24 - 138.$$

Therefore one integer solution of the equation is $x = 6$ and $y = -1$.