Math 255 | Quiz 3 | January 25 2017

Name:

Problem 1: *Show that any integer of the form* $6k+5$ *for* $k \in \mathbb{Z}$ *is also of the form* $3j+2$ *for* $j \in \mathbb{Z}$*, but not conversely.*

Solution: We first tackle the statement

$$n = 6k+5, k \in \mathbb{Z} \implies n = 3j+2, j \in \mathbb{Z}.$$

Suppose that n can be written in the form $6k+5$ for $k \in \mathbb{Z}$. Then

$$\begin{aligned} n &= 6k+5 \\ &= 3(2k) + (3+2) \\ &= 3(2k+1) + 2 \end{aligned}$$

Therefore n can be written in the form $3j+2$ with $j = 2k+1$.
However, the converse, by which we mean the statement

$$n = 3j+2, j \in \mathbb{Z} \implies n = 6k+5, k \in \mathbb{Z},$$

is not true. To show this it suffices to find one counter-example. Our work above suggests that choosing j even, say $j = 2$, will do the trick. Indeed, $n = 8$ can be written as

$$8 = 3 \cdot 2 + 2$$

but there is no $k \in \mathbb{Z}$ with

$$8 = 6k+5.$$