Name:

Problem 1: *Please evaluate this Legendre symbol:*

$$\left(\frac{-219}{383}\right).$$

Note that $219 = 3 \cdot 73$ *and* 383 *is prime. Also you might like to know that* $383 \equiv 18 \pmod{73}$.

Solution: We have that

$$\left(\frac{-219}{383}\right) = \left(\frac{-1}{383}\right)\left(\frac{3}{383}\right)\left(\frac{73}{383}\right).$$

We compute each Legendre symbol separately.

Since $383 \equiv 3 \pmod 4$, $\left(\frac{-1}{383}\right) = -1$.

For the next two symbols we can use Quadratic Reciprocity because the top and bottom are both odd primes. We have

$$\begin{aligned}\left(\frac{3}{383}\right) &= (-1)^{\frac{3-1}{2}\frac{383-1}{2}}\left(\frac{383}{3}\right)\\ &= -\left(\frac{383}{3}\right)\\ &= -\left(\frac{2}{3}\right) = 1,\end{aligned}$$

and

$$\begin{aligned}\left(\frac{73}{383}\right) &= (-1)^{\frac{73-1}{2}\frac{383-1}{2}}\left(\frac{383}{73}\right)\\ &= \left(\frac{383}{73}\right)\\ &= \left(\frac{18}{73}\right)\\ &= \left(\frac{2}{73}\right)\left(\frac{9}{73}\right)\\ &= 1 \cdot 1 = 1,\end{aligned}$$

where $\left(\frac{2}{73}\right) = 1$ because $73 \equiv 1 \pmod 8$.

Therefore

$$\left(\frac{-219}{383}\right) = \left(\frac{-1}{383}\right)\left(\frac{3}{383}\right)\left(\frac{73}{383}\right) = -1.$$