Name:

Problem 1: *Compute each of the following Legendre symbols.*

a) $\left(\dfrac{4}{101}\right)$

Since 4 is a square in the integers, it is a square modulo 101 and $\left(\frac{4}{101}\right) = 1$.

b) $\left(\dfrac{15}{11}\right)$

We cannot apply Quadratic Reciprocity since 15 is not a prime. However, since $15 \equiv 4 \pmod{11}$ and 4 is a square in the integers,

$$\left(\frac{15}{11}\right) = \left(\frac{4}{11}\right) = 1.$$

c) $\left(\dfrac{13}{17}\right)$

Since both 13 and 17 are odd primes, we can apply Quadratic Reciprocity:

$$\left(\frac{13}{17}\right)\left(\frac{17}{13}\right) = (-1)^{\frac{13-1}{2}\frac{17-1}{2}} = 1$$

Therefore

$$\left(\frac{13}{17}\right) = \left(\frac{17}{13}\right) = \left(\frac{4}{13}\right) = 1.$$

The last two equalities are because $4 \equiv 17 \pmod{13}$ and 4 is a square in the integers.