

Name:

Problem 1: *It is a fact that*

$$2^1 \equiv 2 \pmod{4}, \quad 2^2 \equiv 4 \pmod{5}, \quad 2^3 \equiv 3 \pmod{5}, \quad 2^4 \equiv 1 \pmod{5}.$$

What is the index of 4 relative to 2?

For a maximum of two points you may give the definition of the index of a relative to r .

Solution:

If r is a primitive root of n , then the index of a relative to r is the class $\text{ind}_r a$ modulo $\phi(n)$ such that

$$r^{\text{ind}_r a} \equiv a \pmod{n}.$$

Since $4 \equiv 2^2 \pmod{5}$, the index of 4 relative to 2 is 2, the exponent to which 2 must be raised to give 4.