Name:

Problem 1: *Find the units digit of* 3^{100}.
Hint: One way to do it is to use Euler's Theorem, but you may use any technique you like to prove this.

Solution:
Note that this question asks for $3^{100} \pmod{10}$.

Solution 1: Since $\gcd(3, 10) = 1$, we have that

$$3^{\varphi(10)} \equiv 1 \pmod{10}.$$

Since 10 is divisible by 2 and by 5, we have

$$\begin{aligned}\varphi(10) &= 10\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right) \\ &= 10 \cdot \frac{1}{2} \cdot \frac{4}{5} \\ &= 4.\end{aligned}$$

Therefore

$$3^4 \equiv 1 \pmod{10}.$$

Raising both sides to the power of 25 and noticing that $1^{25} = 1$, we get

$$3^{100} \equiv 1 \pmod{10},$$

and the units digit is 1.

Solution 2: We notice that

$$3^2 = 9 \equiv -1 \pmod{10}.$$

Raising both sides to the power of 50 and noticing that $(-1)^{50} = 1$, we get

$$3^{100} \equiv 1 \pmod{10},$$

and again the units digit is 1.