Name:

Problem 1: *What is the order of* 2 *modulo* 7*?*
For a maximum of 2 points, you may instead give the definition of the order of a modulo n, and explain how you could start trying to answer this question.

Solution:
If $n > 1$ and $\gcd(a, n) = 1$ (which is the case if $n = 7$ and $a = 2$), the order of a modulo n is the smallest positive integer k such that $a^k \equiv 1 \pmod{n}$.

Therefore, we are looking for the smallest positive integer k such that $2^k \equiv 1 \pmod{8}$. We can look for this number by successively computing $2 \pmod{7}$, then $2^2 \pmod{7}$, then $2^3 \pmod{7}$, etc. until we get 1 for the first time. The exponent will be the order of 2 modulo 7.

We have that $2 \not\equiv 1 \pmod{7}$ and $2^2 = 4 \not\equiv 1 \pmod{7}$. However, $2^3 = 8 \equiv 1 \pmod{7}$. Therefore the order of 2 modulo 7 is 3.