

Name:

Problem 1: *Please arrange the integers*

$$2, 3, 4, 5, 6, 7, 8, 9$$

in pairs a and b that satisfy

$$ab \equiv 1 \pmod{11}.$$

Solution: We start with the easy pairs: Since $12 \equiv 1 \pmod{11}$, we have

$$2 \cdot 6 \equiv 1 \pmod{11}$$

$$3 \cdot 4 \equiv 1 \pmod{11}.$$

Now we notice that if $ab \equiv 1 \pmod{11}$, then $(-a)(-b) \equiv 1 \pmod{11}$ also. This gives us

$$(-2)(-6) \equiv 9 \cdot 5 \pmod{11}$$

$$(-3)(-4) \equiv 8 \cdot 7 \pmod{11}.$$

And now we are done since we paired up all of the numbers! The pairs are

$$(2, 6)$$

$$(3, 4)$$

$$(5, 9)$$

$$(7, 8).$$