Name:

Problem 1: *Please solve the following set of simultaneous linear congruences:*

$$\begin{aligned} x &\equiv 1 \pmod 3 \\ x &\equiv 2 \pmod 5 \\ x &\equiv 3 \pmod 7. \end{aligned}$$

Solution: This problem is already in the correct form where each equation looks like $x \equiv a_i \pmod{n_i}$ and all of the n_is are pairwise relatively prime. Therefore we can say immediately that

$$\begin{aligned} a_1 = 1, \quad n_1 = 3, \quad \text{and} \quad N_1 = 35 \\ a_2 = 2, \quad n_2 = 5 \quad \text{and} \quad N_2 = 21 \\ a_3 = 3, \quad n_3 = 7 \quad \text{and} \quad N_3 = 15. \end{aligned}$$

We just need to compute the x_is and then we will be ready to put the solution together.

To compute x_1, we must find an integer such that $x_1N_1 \equiv 1 \pmod{n_1}$, or, plugging in our values, $35x_1 \equiv 1 \pmod 3$. Since $35 \equiv 2 \pmod 3$, this is the same as solving $2x_1 \equiv 1 \pmod 3$. By trying $x_1 = 0, 1, 2$ (which are the only distinct values that x_1 can take modulo 3) we see that $x_1 = 2$.

We now compute x_2. We solve $x_2N_2 \equiv 1 \pmod{n_2}$ or $21x_2 \equiv 1 \pmod 5$. Since $21 \equiv 1 \pmod 5$, this is just $x_2 \equiv 1 \pmod 5$, and we can take $x_2 = 1$.

Finally we find x_3. The equation $x_3N_3 \equiv 1 \pmod{n_3}$ here is $15x_3 \equiv 1 \pmod 7$. Since $15 \equiv 1 \pmod 7$, the equation is just $x_3 \equiv 1 \pmod 7$ and we can take $x_3 = 1$.

Now the solution of this simultaneous set of congruences is

$$\begin{aligned} x &\equiv a_1x_1N_1 + a_2x_2N_2 + a_3x_3N_3 \pmod{n_1n_2n_3} \\ &\equiv 1 \cdot 2 \cdot 35 + 2 \cdot 1 \cdot 21 + 3 \cdot 1 \cdot 15 \pmod{3 \cdot 5 \cdot 7} \\ &\equiv 70 + 42 + 45 \pmod{105} \\ &\equiv 157 \pmod{105} \\ &\equiv 52 \pmod{105}. \end{aligned}$$

We can quickly check our answer: Indeed $52 \equiv 1 \pmod 3$, $52 \equiv 2 \pmod 5$ and $52 \equiv 3 \pmod 7$.