MATH 255 QUIZ 10 FEBRUARY 17 2017

Name:

Problem 1: *Please solve the following linear congruence:*

$$6x \equiv 15 \pmod{21}.$$

Solution: Here we have $a = 6$, $b = 15$ and $n = 21$. In addition, it is possible to see, without doing the Euclidean algorithm, that $\gcd(6, 21) = 3$. Since 3 divides 15, this linear congruence has at least one solution, and in fact it has $\gcd(a, n) = 3$ solutions.

We adopt the approach explained in class on Wednesday to solve the equation:

1. We already found $\gcd(a, n)$ and determined that there was a solution.

2. We first find one solution to the equation $6x + 21y = 3$. By inspection, this has solution $x_0 = -3$ and $y_0 = 1$.

3. This gives us a particular solution for the equation $6x + 21y = 15$: $x_p = -15$ and $y_p = 5$.

4. All integer solutions are of the form

$$x = -15 + 7t$$
$$y = 5 - 2t$$

   for $t \in \mathbb{Z}$.

5. We now forget about y, and give the three different values that x can take modulo 21. They are

$$x = -15 + 0 \equiv 6 \pmod{21} \quad \text{(this is when } t = 0\text{)}$$
$$x = -15 + 7 = -8 \equiv 13 \pmod{21} \quad \text{(this is when } t = 1\text{)}$$
$$x = -15 + 14 = -1 \equiv 20 \pmod{21} \quad \text{(this is when } t = 2\text{)}.$$

   (Note that if we continue and take $t = 3$, we get $x = -15+21 \equiv 6 \pmod{21}$. This is not a new solution, and so we know that we got all of the possible solutions.)

The solutions of the congruence are $x \equiv 6, 13$ or $20 \pmod{21}$.