Math 255 - Spring 2017
Homework 7

This homework is due on Wednesday, March 29 by 5pm. Please support every assertion that you make with either a precise reference from the textbook (theorem number or page) or provide a proof.

1. The *Mangoldt function* Λ is defined by

$$\Lambda(n) = \begin{cases} \log p & \text{if } n = p^k \text{, where } p \text{ is a prime and } k \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Prove that

$$\log(n) = \sum_{d|n} \Lambda(d).$$

   (b) Use part (a) to prove that

$$\Lambda(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) \log d = -\sum_{d|n} \mu(d) \log d.$$

   (You must prove both equalities in this statement.)

2. Let

$$S(n) = \#\{d : d|n \text{ and there is no prime } p \text{ with } p^2|d\},$$

   where # denotes the cardinality of a set.

   (a) Prove that $S(n) = \sum_{d|n} |\mu(d)|$.

   (b) Prove that $S(n)$ is multiplicative.

   (c) Prove that $S(n) = 2^{\omega(n)}$, where

$$\omega(n) = \#\{p : p|n \text{ and } p \text{ is prime}\}.$$

3. Prove the following:

   (a) If n is odd, then $\phi(2n) = \phi(n)$.

   (b) If n is even, then $\phi(2n) = 2\phi(n)$.