

Math 255 - Spring 2017  
Homework 6

This homework is due on Friday, March 24 by 5pm. Please support every assertion that you make with either a precise reference from the textbook (theorem number or page) or provide a proof.

1. Show that

$$18! \equiv -1 \pmod{437}$$

Hint:  $437 = 19 \cdot 23$

2. If  $n$  is a composite integer, show that  $(n - 1)! \equiv 0 \pmod{n}$ , except when  $n = 4$ .
3. For any integer  $n \geq 1$ , establish the inequality  $\tau(n) \leq 2\sqrt{n}$ .
4. (a) Show that if  $f$  is multiplicative, then  $f^k$  (where  $f^k$  means the function that sends  $n$  to  $f(n)$  raised to the  $k$ th power) is also multiplicative.
- (b) Show that the function

$$F(n) = \sum_{d|n} \tau(d)^3$$

is multiplicative.

- (c) Show that the function

$$G(n) = \left( \sum_{d|n} \tau(d) \right)^2$$

is multiplicative.

- (d) Let  $f$  and  $g$  be multiplicative functions that are not identically zero. Show that if  $f(p^k) = g(p^k)$  for all primes  $p$  and all integers  $k \geq 1$ , then  $f(n) = g(n)$  for all  $n \geq 1$ .
- (e) Show that for all  $n \geq 1$ ,  $F(n) = G(n)$ , where  $F$  and  $G$  are as defined in parts (b) and (c) of this problem.  
Hint: Use part (d).