Math 255 - Spring 2017
Homework 5

This homework is due on Monday, February 27. Please support every assertion that you make with either a precise reference from the textbook (theorem number or page) or provide a proof.

1. Find the smallest integer $a > 2$ such that

$$2|a, \quad 3|(a+1), \quad 4|(a+2), \quad 5|(a+3), \quad \text{and} \quad 6|(a+4).$$

2. This is an ancient Chinese problem: A band of 17 pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal distribution left 10 coins. Again an argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What was the least number of coins that could have been stolen?

3. If $\gcd(a, 35) = 1$, show that $a^{12} \equiv 1 \pmod{35}$.
Hint: Use Fermat's Little Theorem **and** the Chinese Remainder Theorem.