Math 255 - Spring 2017
Homework 3

This homework is due on Monday, February 6.

1. Prove that if $a, b \in \mathbb{Z}$, then

$$ab = \operatorname{lcm}(a, b) \gcd(a, b),$$

where the least common multiple (lcm) is as in Definition 2.4 (page 29).

2. Prove that the greatest common divisor of two positive integers divides their least common multiple.

3. Assuming that $\gcd(a, b) = 1$, prove that $\gcd(a + b, a - b) = 1$ or 2.

4. This puzzle is due to Yen Kung (1372): You have an unknown number of coins. If you divide the coins into 77 piles, you are 50 coins short; but if you divide the coins into 78 piles, the remainder is zero. How many coins do you have?