Math 255 - Spring 2017
Homework 11

This homework is due on Friday, May 5 by 5pm. Please support every assertion that you make with either a precise reference from the textbook (theorem number or page) or provide a proof.

1. Give all solutions to the equation $x^2 \equiv 7 \pmod{63}$.

2. Give all solutions to the following quadratic equations:

   (a) $x^2 + 5x + 6 \equiv 0 \pmod{5^3}$

   (b) $x^2 + x + 3 \equiv 0 \pmod{3^3}$

3. For $n > 1$, let $f(n)$ be the number of solutions to the equation $x^2 \equiv 1 \pmod{n}$, and let $\omega(n)$ be the number of distinct primes dividing n.

   (a) Give a closed formula for $f(n)$. Your formula should use $\omega(n)$.

   (b) In the solutions to Homework 8, problem 1, it is erroneously stated that $f(n)$ is always even. When is $f(n)$ odd?

   (c) Assume that $f(n)$ is even. In the solutions to Homework 8, problem 1, we explain that
   $$\prod_{a \in (\mathbb{Z}/n\mathbb{Z})^\times} a \equiv (-1)^{f(n)/2} \pmod{n}.$$
   Assuming that $f(n)$ is even, when is
   $$\prod_{a \in (\mathbb{Z}/n\mathbb{Z})^\times} a \equiv -1 \pmod{n}?$$