Math 255 - Spring 2017
Homework 1

This homework is due on Monday, January 23.

1. Show that the expression
$$\frac{(2n)!}{2^n n!}$$
is an integer for all $n \geq 0$.

2. Prove that if $n \geq 1$,
$$\binom{n}{r} < \binom{n}{r+1}$$
if and only if
$$0 \leq r < \frac{1}{2}(n-1).$$
Note: This is an "if and only if" statement, so you must prove both implications.

3. In the following puzzle, each of the ten letters appearing stands for exactly one digit, with no repeats. If the assignment of digits to letters is such that the addition problem

$$\begin{array}{r} \text{GERALD} \\ + \ \text{DONALD} \\ \hline \text{ROBERT} \end{array}$$

is correct and furthermore we know that $D = 5$, deduce the value of each of the other letters.
Note: This is not an induction proof. For full credit, you must *reason* your way to the value of each letter (i.e. no guessing).