Abstract Algebra III

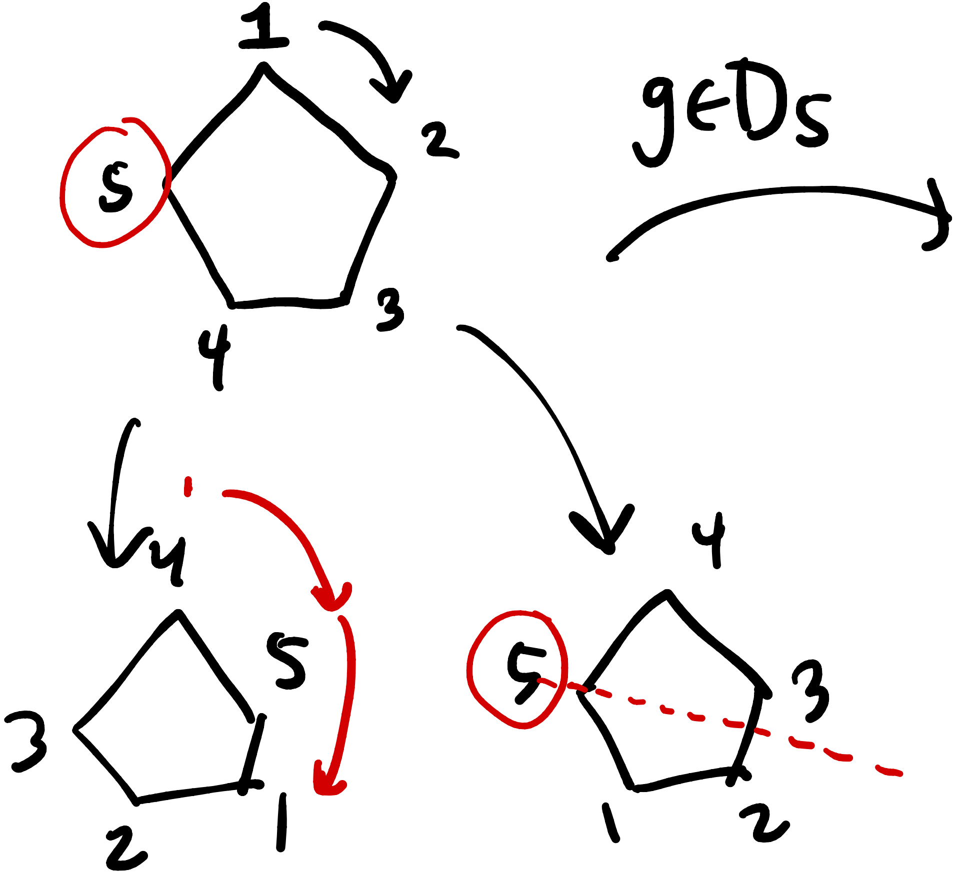
This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

"First" D_4 is $D_4 \times \{1\} \ni \{(g,1) : g \in D_4\}$

D_n is the symmetries of the regular n gon

2 kinds: Rotations

Reflections

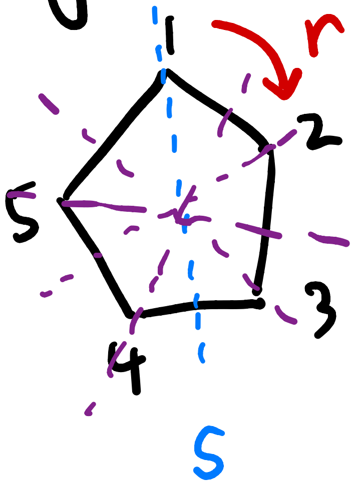


Rotation! Reflection

g is a rotation if after I do it, 2 is still clockwise from 1

g is a reflection if afterwards 2 is anticlockwise from 1

Why do I say that



Truefact: D_5 is generated by r and s

Elements of D_5 are

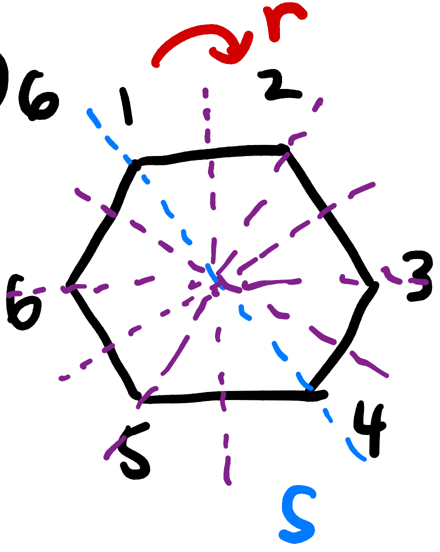
$1, \ r, \ r^2, \ r^3, \ r^4, \ \cancel{r^5=1}$

$s, \ \boxed{sr, \ sr^2, \ sr^3, \ sr^4}$

$\cancel{s^2=1}$

all reflections!

D_6



$1, r, r^2, r^3, r^4, r^5, \cancel{r^6 = 1}$

$s, \boxed{sr, sr^2, sr^3, sr^4, sr^4}$

$\cancel{s^2 = 1}$

Reflections also!

D_n always has $n-1$ rotations r^i

n reflections sr^j

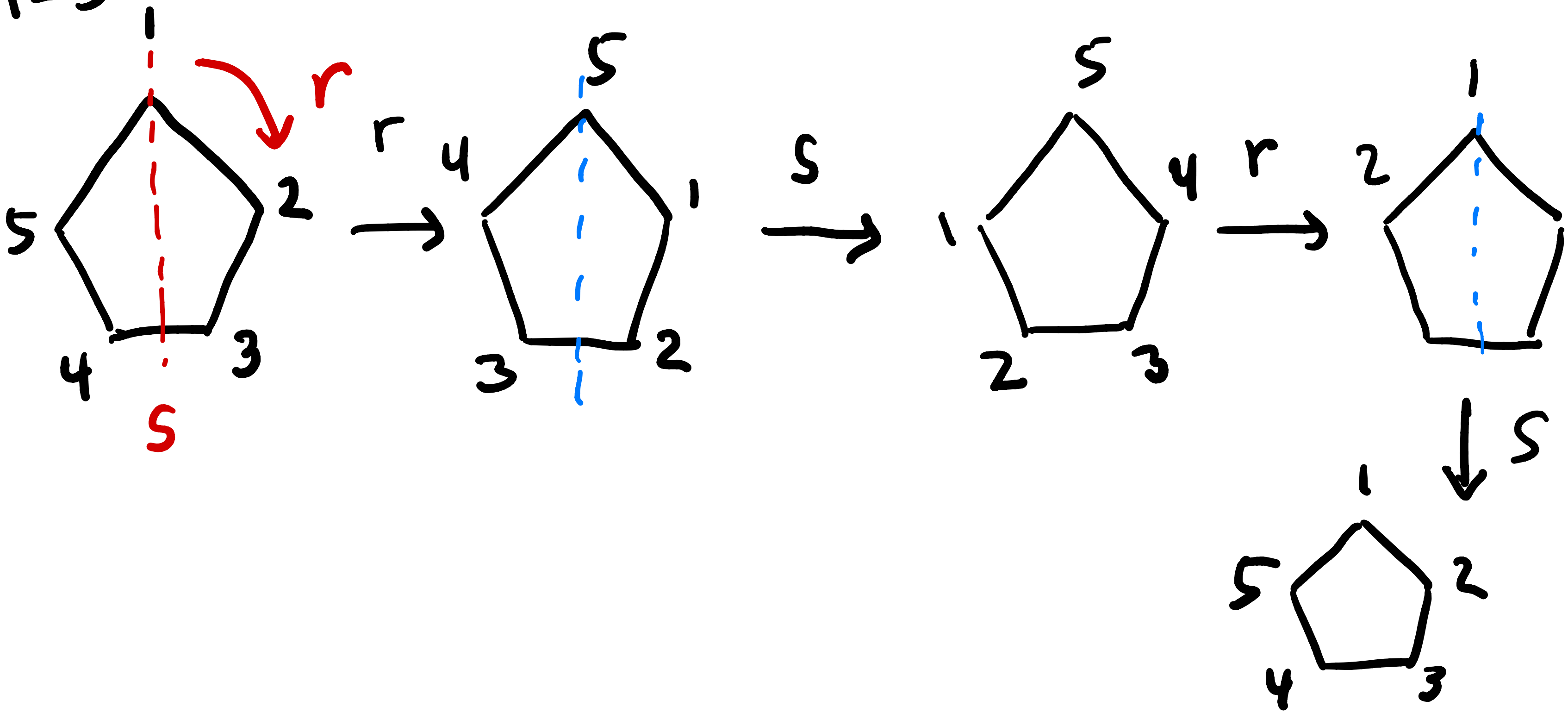
or $r^k s$

So in $D_n \ni r, s$

always true that

$rsrs = 1$

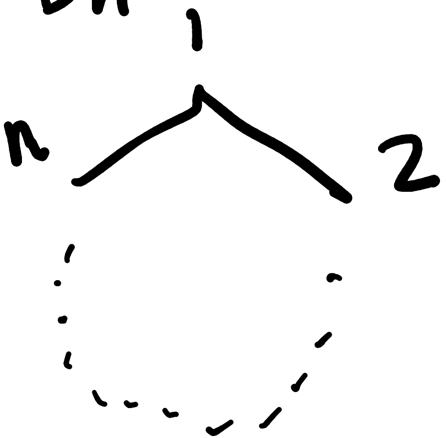
$n=5$



Why is $rsrs=1$ always?

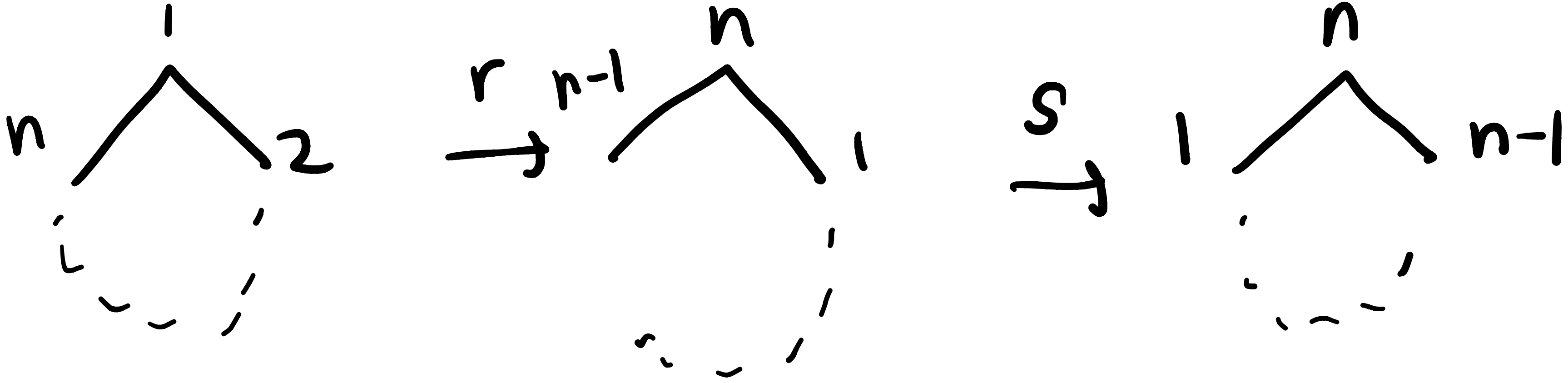
Track where 1 is and the orientation

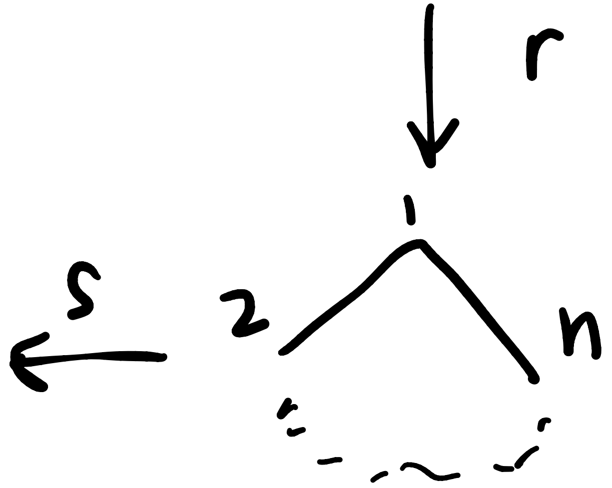
D_n



r moves 1 one unit to the right, still clockwise

s moves 1 to the position one unit to the left of original position, now counter clockwise

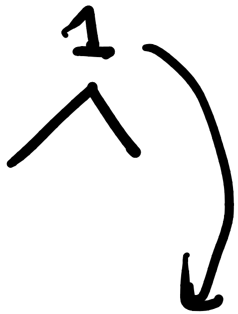




r brings 1 back to the top
but still anticlockwise

s fixes 1 in its spot
now clockwise

In the chat: Also $r^i s r^i s = 1$



r^i does "to the left" by i units

then s puts 1 "to the right" by i units

then r^i brings 1 back to the top

then s fixes orientation

$$r^i s r^i s = r^{i-1}(rs)r^i s = r^{i-1}(sr^{-1})r^i s$$
$$= r^{i-1} s r^{i-1} s$$

$rsrs = 1 \Rightarrow rs = sr^{-1}$

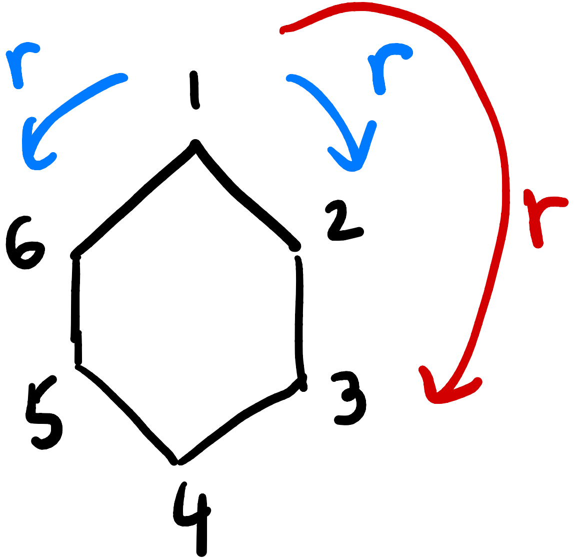
$rs = s^{-1}r^{-1}$

but $s^{-1} = s$

$$= r^{i-2}(rs)r^{i-1}s$$
$$= r^{i-2}(sr^{-1})r^{i-1}s$$
$$= r^{i-2} s r^{i-2} s$$
$$\vdots$$
$$= s^2 = 1$$

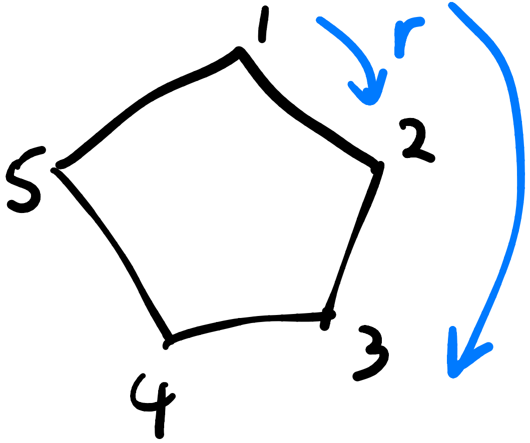
Still true that D_n contains

$1, r, r^2, \ldots r^{n-1}$



← this cannot be my choice of "fundamental rotation"

But in D_5



For more on presentations p25 of D&F

$$Y = \langle u, v \mid u^4 = v^3 = 1,\ uv = v^2u^2 \rangle = 1$$

(p.27)

$$D_n = \langle r, s \mid r^n = 1,\ s^2 = 1,\ rsrs = 1 \rangle$$

In $D_4 \times D_3$ Find
- $R = (g_1, g_2)$ $\quad R^4 = 1$

  $R^i \neq 1 \quad i = 1, 2, 3$

$$\begin{gathered} \cup\!| \\ G = \langle R, S \rangle \\ \| 2 \\ D_4 \end{gathered}$$

- $S = (h_1, h_2)$ $\quad S^2 = 1$

- $RSRS = 1$

$R = (g_1, g_2)$

$g_1 \in D_4$

$g_2 \in D_3$

ASIDE

If $\sigma^4 = 1$

then order of σ is

1, 2, 4

$R^4 = 1$

$= (g_1^4, g_2^4) = (1, 1)$

$\Rightarrow g_2 = 1$

g_2 reflection

$\Rightarrow g_1$ has order 4

$S = (h_1, h_2)$

$$RSRS = 1$$

$$\begin{array}{c} \text{\textcolor{red}{$=1$}} \\ \rotatebox{90}{$=$} \end{array}$$

$$(g_1 h_1 g_1 h_1 \ , \ g_2 h_2 g_2 h_2)$$

$$\begin{array}{cc} \shortparallel & \shortparallel \\ 1 & 1 \end{array}$$

$g_1 = r$ ✓

$h_1 = s$

$\cancel{g_2 = 1}$

$\cancel{h_2 = 1}$

$\cancel{g_2 = r}$

$\cancel{h_2 = s}$

$\Rightarrow$ $R = (r, r)$

order 12

2 ways:

Pick 8 elements & try to show...

D_4

$sr^2 = s$

$SRR = RSR$

$rsr = s$

D_4 $(1,1) \longmapsto 1$

$\boxed{{}^{R}(r,s)} \longmapsto r$ ← order 4

$\boxed{{}^{S}(s,s)} \longmapsto s$

$1 \quad R^2=(r^2,1) \quad R^3=(r^3,s)$

RSRS=1

$S=(s,s) \quad RS=(rs,1) \quad R^2S=(r^2s,s)$

$R^3S=(r^3s,1)$

$(r,s)(r,s)(r,s)(s,s)$

$$\boxed{\begin{aligned} R &= (r, rs) \\ S &= (s, rs) \end{aligned}}$$

$R = (r, s)$

$S = (s, s)$

$R = (r, r^2 s)$

$S = (r, r^2 s)$

That’s all for today!