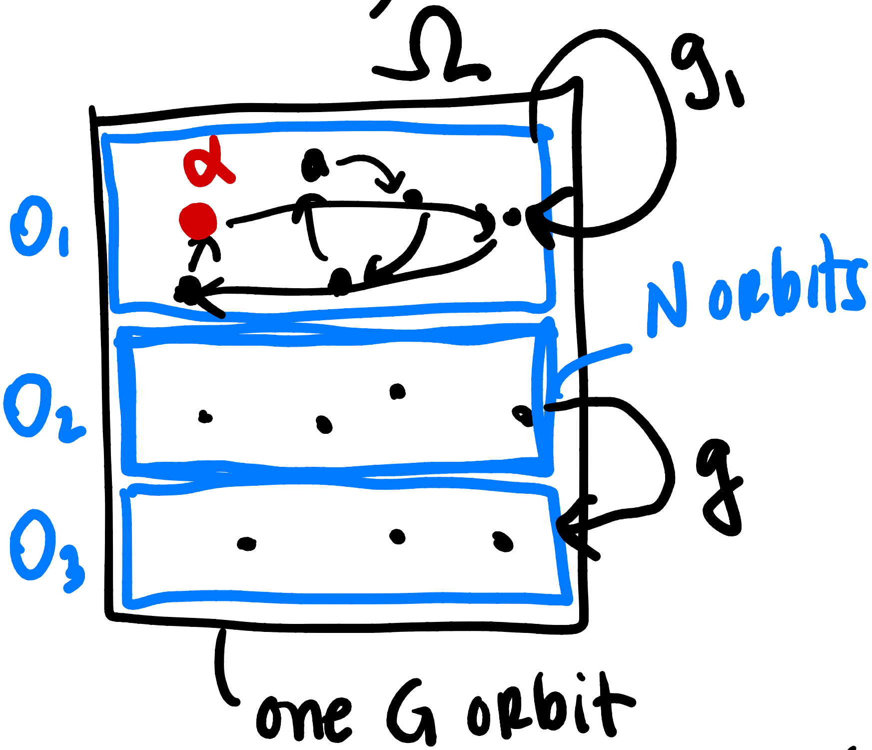
Quiz is available on Teams but maybe wait?

Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

HW 3 #1c)



~~r = # of N-orbits in Ω~~

$G \curvearrowright$ N-orbits transitively

r = # of elements in the G-orbit of O_1

$$r = [G : \mathrm{Stab}_{O_1}]$$

Orbit-stabilizer theorem

Want to show $r = [G : NG_\alpha]$

Enough to show $\mathrm{Stab}_{O_1} = NG_\alpha$

i.e. if $n \in N$ and $g \in G_\alpha$ then $ng \in \mathrm{Stab}_{O_1}$ 1

& if $h \in \mathrm{Stab}_{O_1}$ then $\exists\, n \in N,\ g \in G_\alpha$ 2

with $h = ng$

to show 1 it is enough to show that for $\alpha \in \mathcal{O}_1$,

$$(ng)\cdot\alpha \in \mathcal{O}_1$$

this is an N-orbit

$$(ng)\cdot\alpha = n\cdot(g\cdot\alpha) = n\cdot\alpha \in \mathcal{O}_1$$

$g \in G_\alpha$

Let $\alpha \in \mathcal{O}_1$ and $h\mathcal{O}_1 = \mathcal{O}_1$

it means that $h\cdot\alpha \in \mathcal{O}_1$

Goal: Show

$h = ng$

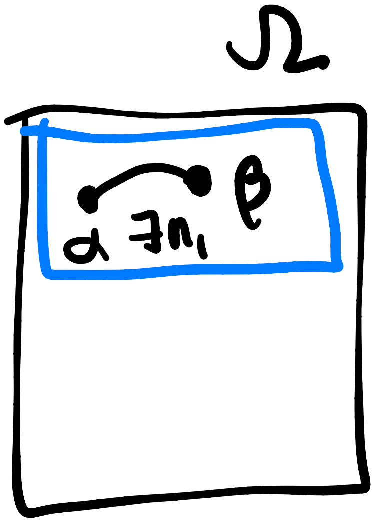
$n \in N$ $g \in G_\alpha$

if $h \in N$, done (use $g = 1$)

could $h \notin N$?? If not $\exists\, n_1 \in N$ with

$$h \cdot \underbrace{\alpha}_{=\beta \in O_1} = n_1 \cdot \alpha$$

O_1



$$\Rightarrow (n_1^{-1} h) \cdot \alpha = \alpha$$

so $n_1^{-1} h \in G_\alpha$

say $n_1^{-1} h = g \in G_\alpha$ so $h = n_1 g$ ✓

Now: Quiz 3

That's all for today!

See you on Campuswire

(HW 3 due @ midnight
on Gradescope)