Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Today: open questions

Wednesday: HW4 #6

Friday: open questions

then $\frac{\text{Quiz 3}}{\text{HW3}}$

HW 3 #6

If $H \trianglelefteq G$, and H and G/H is solvable then G is solvable.

- You should be able to prove this
- For 6d) you don't need to prove it, can just use it.

6d) If G is a <u>p-group</u> then G is solvable

$\# G = p^{\alpha}$ $\quad$ p prime $\quad$ $\alpha \geq 1$

By induction on α.

Base case: If $\alpha = 1$, $\# G = p \Rightarrow G \cong C_p$

$1 \trianglelefteq G \quad G/1 \cong C_p$ cyclic

Strong induction: Assume that if $\# G = p^i$ for $1 \leq i \leq n$ then G is solvable.

Let $\# G = p^{n+1}$

Let $H = Z(G) \trianglelefteq G$, by (b) $Z(G) \neq 1$

If $Z(G) = G$, G abelian hence solvable

Otherwise $\# Z(G) = p^{\beta}$ $1 \leq \beta < n+1$

and $\# G/Z(G) = p^{\gamma} \quad 1 \leq \gamma < n+1$

$$\frac{\# G}{\# Z(G)} = \frac{p^{n+1}}{p^{\beta}} = p^{n+1-\beta}$$

By strong induction both $Z(G)$ and $G/Z(G)$ are soluable, so by this theorem, G is solvable.

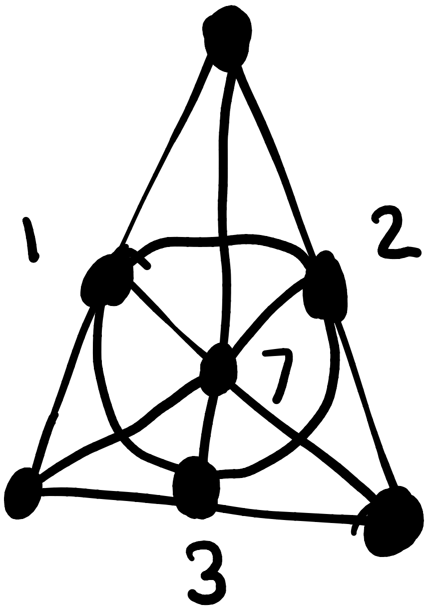
HW3 #1

$O_1, O_2 \dots, O_r$ orbits of N acting on Ω

so $r :=$ # orbits of N acting on Ω

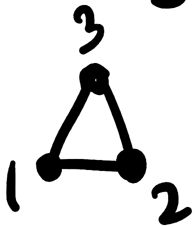
show $= [G : NG_\alpha]$

HW3 #3



7 is fixed

$D_3 \cong S_3$

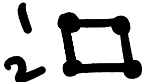


$\{1, 2, 3\}$

$A_3 \cong C_3$

8 24

$D_4 \not\cong S_4$



$\{1, 2, 3, 4\}$

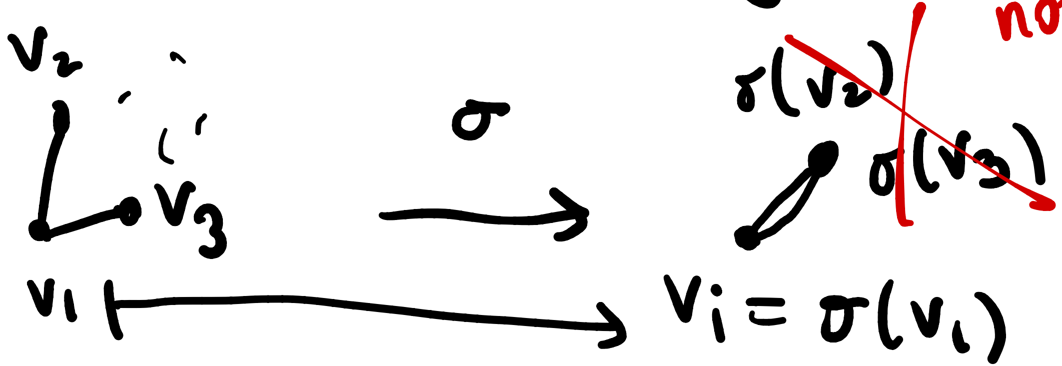
A graph automorphism must preserve degree

i.e. if $\sigma(v_i) = v_j$

then $\deg v_i = \deg v_j$

$\quad\quad\quad\;\, \|$

\# edges touching v_i



not bijective on vertices

Should be able to show that a graph aut

- is a bijection on edges
- preserves degree $\quad \deg \sigma(v_i) = \deg v_i \quad \forall i$

But also can use these facts without proof when needed.

Graph aut: permutation of vertices

↳ bijection from a set to itself

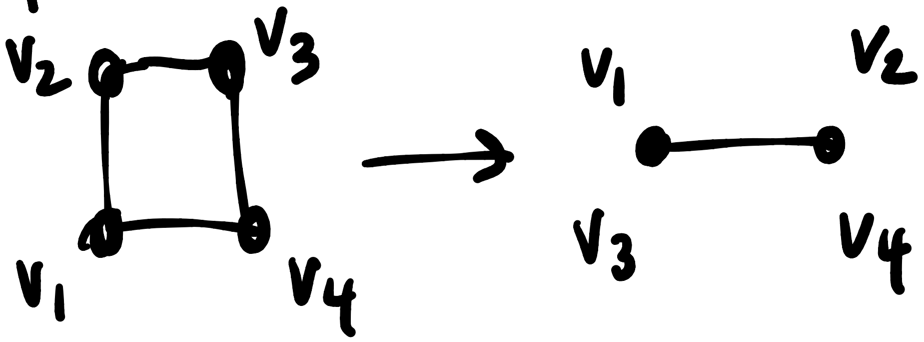
sends edges to edges

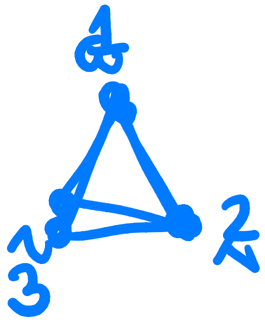
if $\exists$ edge from v_i to v_j

then $\exists$ edge from $\sigma(v_i)$ to $\sigma(v_j)$

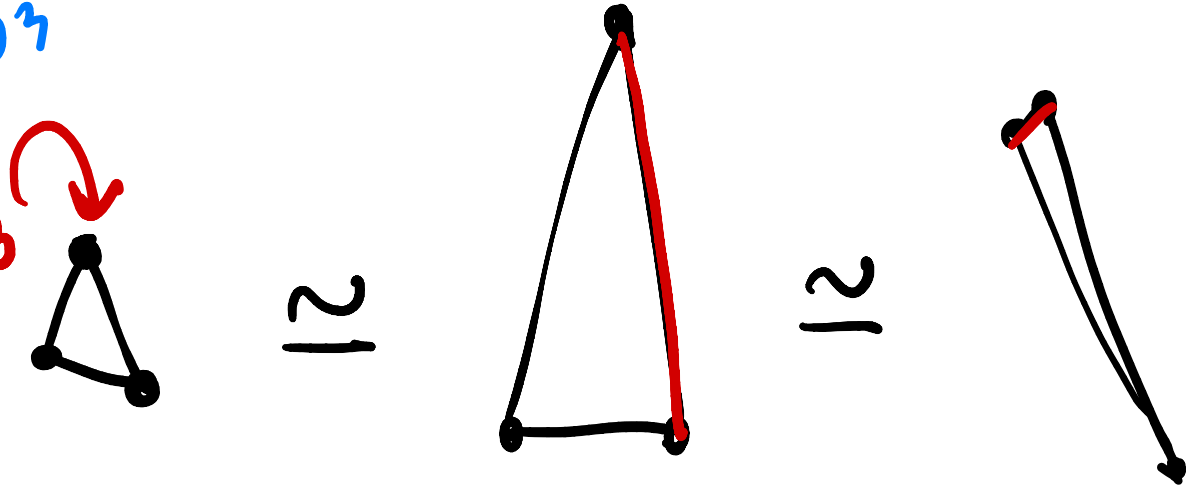
graph aut is also permutation of edges

Graph morphism





$S_3 \cong D_3$



Office hours

M noon - 1pm

W 3pm - 5pm

Post on Campuswire

That’s all for today!