Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

No quiz today: Quiz 3 next Friday

Next Wednesday: Focus on HW4 Problem 6

- look up all the definition
- theorems
- try solving the problem

Due dates: HW2 due on Gradescope tonight

HW3 give peer reviewer(s) today

Campuswire: forum to post questions

Class code is 9234

HW3 #3 c) $G \cong D_3 \Rightarrow \#G = 6$

$$\Rightarrow \underbrace{5,7 \nmid \#G}_{b)}$$

Another way to prove this is by contradiction

Let $p = 5$ or 7 (both primes)

Suppose for a contradiction that $p \mid \#G$

then because p is a prime, by Cauchy's Thm

there is an element of G of order p, say $g \in G$

has order p.

But then since the action of G on the graph is faithful, this would mean that there is v with

$$g\cdot v \neq g^2\cdot v \neq g^3\cdot v \neq \cdots \neq g^{p-1}\cdot v \neq v$$

- no 2 elements act the same way
- $\varphi: G \to S_7$ permutation rep is injective

$\forall\, g_1 \neq g_2 \in G \quad \exists\, v$ s.t. $g_1(v) \neq g_2(v)$

this would imply that G has an orbit of size at least p (contains $\{v, g\cdot v, g^2\cdot v, \ldots, g^{p-1}\cdot v\}$

But we know from part a) the largest orbit has size 3.

Important action is faithful

if $\exists\, i \neq j$ with $g^i \cdot v = g^j \cdot v \ \forall v \in V$

Important thing for qual:

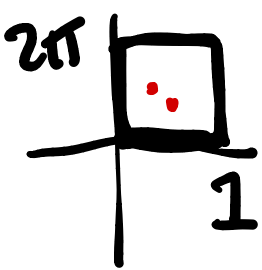
Know all definitions + theorems

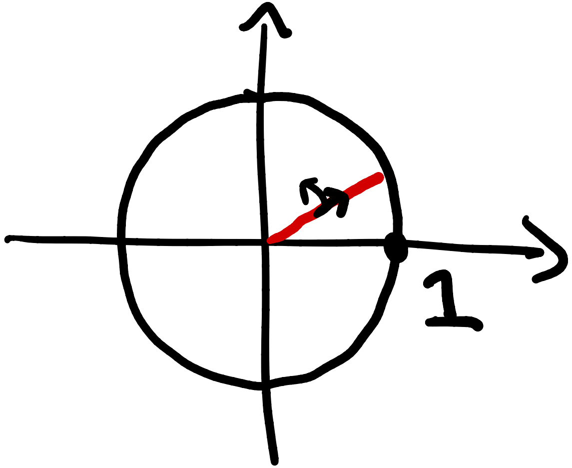
Write them down as answers when stuck

Caution: A false definition or theorem or fact on the qual can sink you.

$$re^{i\phi} \longmapsto r + i\phi$$

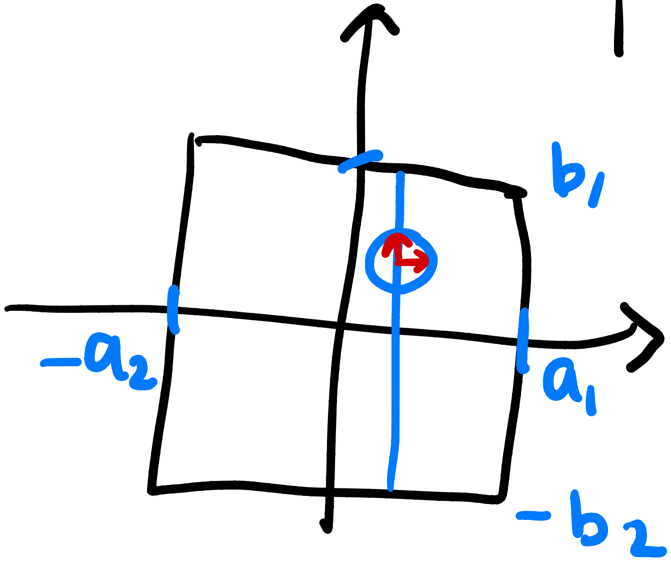
$$z = re^{i\phi}$$





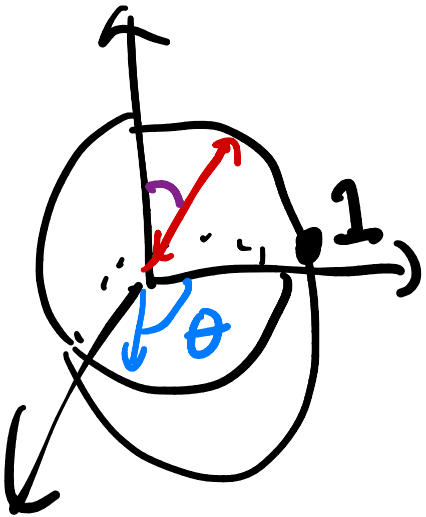
$$0 \le r \le 1$$

$$0 \le \phi \le 2\pi$$



$$-a_2 \le x \le a_1$$

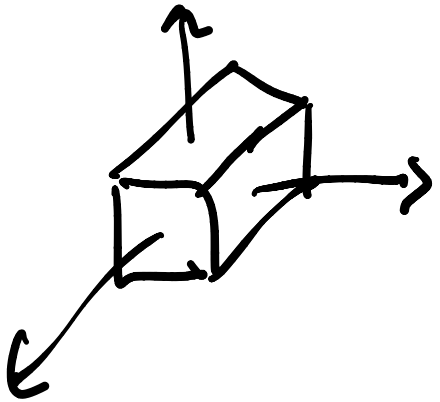
$$-b_2 \le y \le b_1$$



$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi$

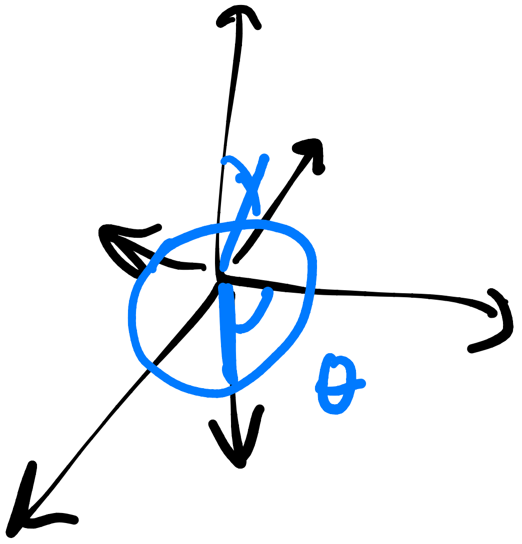


$a_1 \leq x \leq a_2$

$b_1 \leq y \leq b_2$

$c_1 \leq z \leq c_2$

No matter dimension a ball is everything less than R away from 0



HW3 # 2 c) $G/C_G(N) \leq \operatorname{Aut}(N)$

More generally true:

Always true that $G/Z(G) \leq \operatorname{Aut}(G)$

$Z(G) = C_G(G)$

Why: Have $G \curvearrowright G$ by conjugation

Show that conjugation by g is an automorphism

i.e. fix $g \in G$

$$\phi_g : \begin{array}{l} G \longrightarrow G \\ x \longmapsto gxg^{-1} \end{array}$$

- bijective
- homomorphism

$\Downarrow$

$$\Phi : \begin{array}{l} G \longrightarrow \mathrm{Aut}(G) \\ g \longmapsto \phi_g \end{array}$$

$$\Phi : G \to \mathrm{Aut}(G)$$
$$g \mapsto \phi_g$$

By 1^{st} isom thm

$$G/\overset{Z(G)}{\cancel{\ker \Phi}} \cong \mathrm{Im}\,\Phi \leq \mathrm{Aut}(G)$$

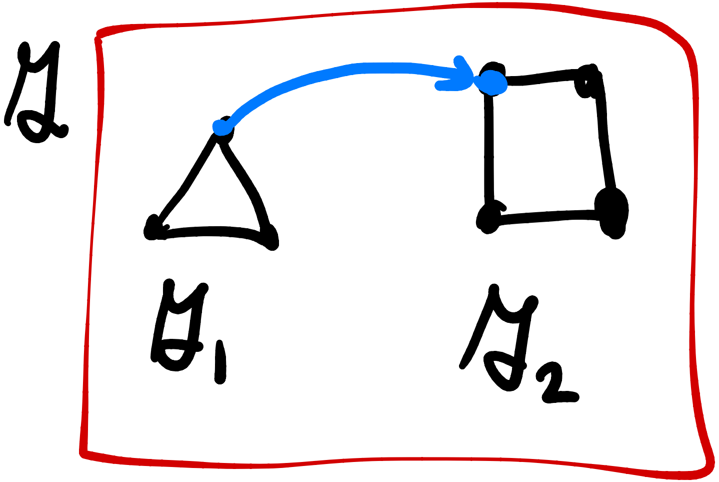
$\ker \Phi$: this is $g \in G$ with $\phi_g = id$

$$\phi_g(x) = x \quad \forall\, x \in G$$

$$gxg^{-1} = x \iff gx = xg \iff g \in C(G)$$

the elements of $G/Z(G)$ are the "inner" auts of G

$$\begin{array}{c} G/Z(G) \\ \rotatebox{90}{$\neq$} \\ \mathrm{Aut}(G) \end{array}$$



$$\mathrm{Aut}(\mathcal{G}) = \mathrm{Aut}(\mathcal{G}_1) \times \mathrm{Aut}(\mathcal{G}_2)$$

$$\Updownarrow$$

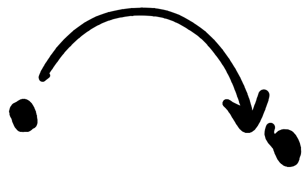
$$\psi$$

$$\sigma = (\sigma_1, \sigma_2)$$

$$\tau = (\tau_1, \tau_2)$$

$$\sigma\tau = (\sigma_1\tau_1, \sigma_2\tau_2)$$

$\mathcal{G}$



$\mathcal{G}_1$ $\mathcal{G}_2$

suppose that

$\sigma \in \mathrm{Aut}(\mathcal{G})$

with $\sigma(v) \in \mathcal{G}_2$ for

$v \in \mathcal{G}_1$

$\Rightarrow \sigma$ interchanges $\mathcal{G}_1$ & $\mathcal{G}_2$

$\Rightarrow \sigma$ isomorphism between

$\mathcal{G}_1$ and $\mathcal{G}_2$

That’s all for today!