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# Abstract Algebra III

— This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat. —

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Questions?

→ some good stuff on Campuswire

Commutator Subgp

$G$  a finite gp

define commutator subgp

$$G' = \left\langle \underbrace{\{ ghg^{-1}h^{-1} : g, h \in G \}} \right\rangle$$

group  
generated  
by

The failure of  $g, h$  to commute is measured

by  $ghg^{-1}h^{-1}$

i.e. if  $ghg^{-1}h^{-1} = 1$  then they commute

and if not, then they don't,

$gh = hg$

$G' = 1 \Rightarrow G$  abelian

If  $G'$  is big

then far from ab

If  $G/N$  is abelian then we must have that

$$ghg^{-1}h^{-1} \in N \quad \forall g, h \in G$$

Another way to phrase it:

$$G/N \text{ abelian} \quad \text{iff} \quad G' < N$$

If  $\varphi: G \rightarrow A$ ,  $\varphi$  gp hom,  $A$  abelian

then  $\varphi$  "factors through"  $G/G'$  i.e.

$$\begin{array}{ccc} \varphi: G & \longrightarrow & A \\ & \searrow \cong \nearrow & \\ & G/G' & \end{array}$$

$$G/\ker \varphi \cong \overset{\text{Im } \varphi}{B} < A$$

abelian

$$\Rightarrow G' = \ker \varphi$$

More about this on Campuswide

Quiz 4

upload to Gradescope

That's all for today!