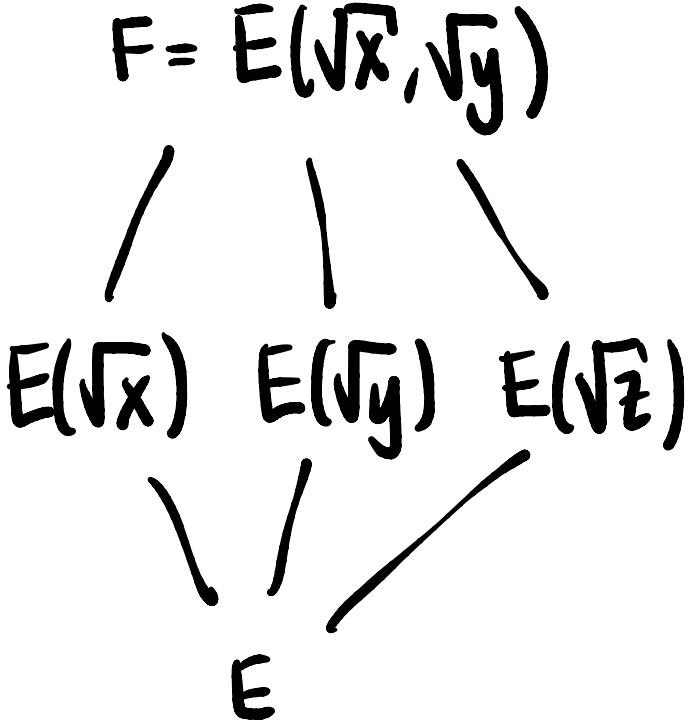
Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

HW9 #4 $\quad \mathrm{Gal}(F/E) \cong C_2 \times C_2 \cong V_4$



x, y not squares in E

Last thing to show is that

xy not square in E

$E(\sqrt{xy}) \neq E(\sqrt{x})$

$E(\sqrt{xy}) \neq E(\sqrt{y})$

not needed to finish

If xy is a square in E, say $xy = a^2$ $a \in E$

then $\sqrt{x} = a \cdot \frac{1}{\sqrt{y}}$ $\rightarrow$ $\sqrt{x} \in E(\sqrt{y})$

$\Rightarrow$ $E(\sqrt{x}) = E(\sqrt{y})$

false

so xy is not a square in E

$$\left[E(\sqrt{xy}) : E\right] = 2$$

So this is one implication

$[F:E] = 4$

$\mathrm{Gal}(F/E) \cong C_2 \times C_2$

$\Rightarrow$

$\exists\, x, y \in E$, x, y, xy nonsquares

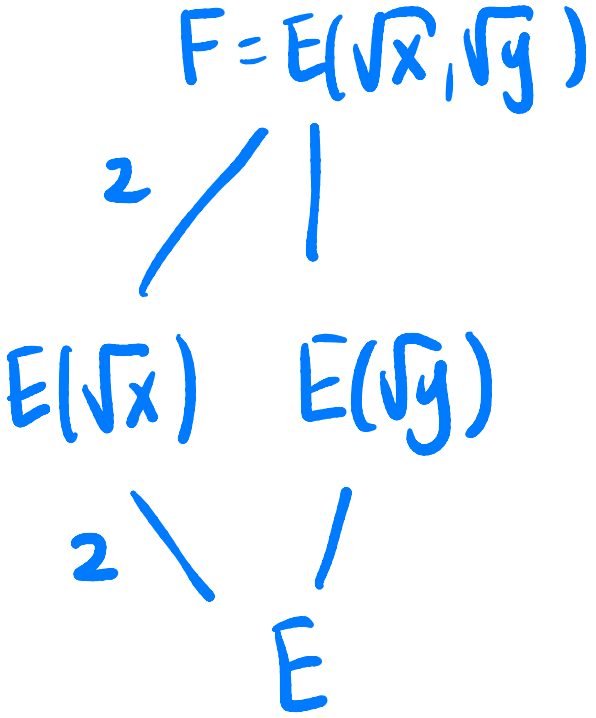
$F = E(\sqrt{x}, \sqrt{y})$

$\Leftarrow$

If $F = E(\sqrt{x}, \sqrt{y})$ x, y not squares in E

then $E(\sqrt{x}), E(\sqrt{y}) \subseteq F$

and $[E(\sqrt{x}):E] = [E(\sqrt{y}):E] = 2$



* $E(\sqrt{x}) \neq E(\sqrt{y})$ because xy is not a square (justify this briefly)

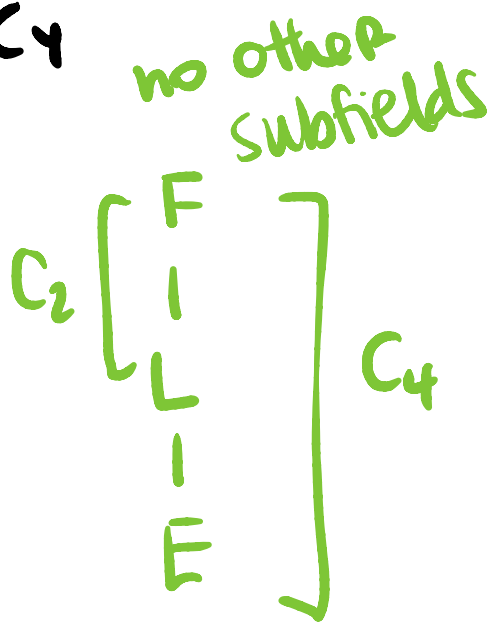
copy knew this

(* $[F:E] = 4 = [F:E(\sqrt{x})] \cdot [E(\sqrt{x}):E]$)

Difference between C_4 and $C_2 \times C_4$

subgps of C_4 are exactly

$\langle g \rangle$ $\quad$ $1, C_2, C_4$ $\quad$ $\langle g^2 \rangle$



Fact: C_n has exactly 1 subgp (cyclic) of order d for each $d \mid n$, and no more subgps.

Last thing to finish: F/E is Galois

when $F = E(\sqrt{x}, \sqrt{y})$

Quiz time!

HW 8 due

That's all for today!