Quiz 10

Abstract Algebra III

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

We know that any abelian extension of $\mathbb{Q}$ is contained in a cyclotomic extension

is a field extension of $\mathbb{Q}$ generated by the torsion points of a $\mathbb{Z}$-action on $\mathbb{C}$

$(\mathbb{Z}, +)$

one way that $\underbrace{\mathbb{Z} \circlearrowright \mathbb{C}}$

$\mathbb{Z}$ acts on $\mathbb{C}$

$\underset{\mathbb{Z}}{\underset{\uparrow}{n}} \cdot \underset{\mathbb{C}}{\underset{\uparrow}{z}} \rightsquigarrow$ new number in $\mathbb{C}$

1st action of $\mathbb{Z}$ on $\mathbb{C}$ is multiplication:

$$n \cdot z = nz$$

torsion of this action is all the numbers $z \in \mathbb{C}$ s.t. $\exists\, n$ with $n \cdot z = 0$

$z = 0$

Another action of $\mathbb{Z}$ on $\mathbb{C}$ is $\quad n \cdot z = z^n$

torsion of this action is $z \in \mathbb{C}$ s.t.

$\exists n$ with $z^n = 1$

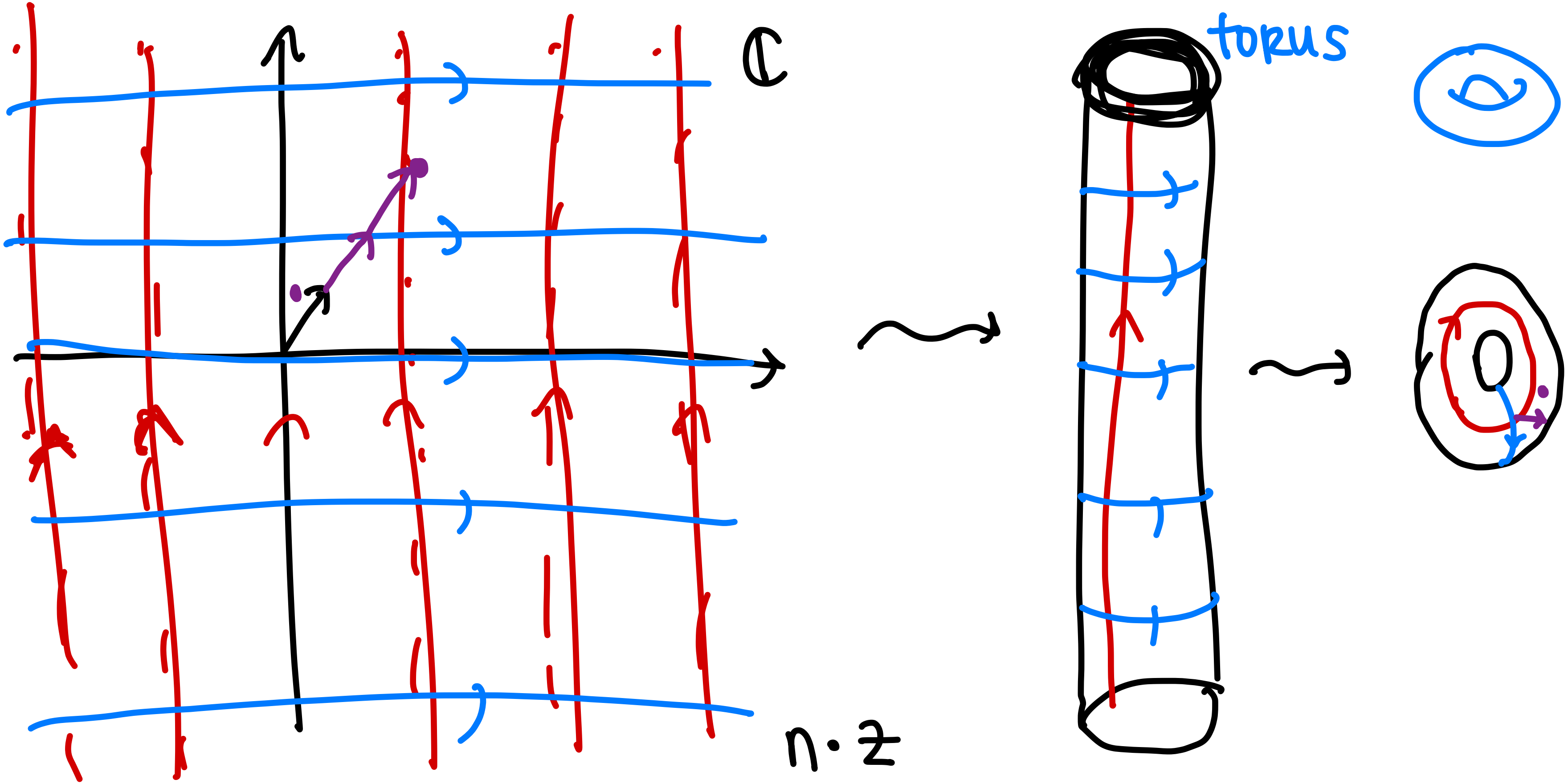
$z = 1$

the roots of unity!

↳ generate the cyclotomic extensions

↳ all the abelian extensions of $\mathbb{Q}$ are in cyclotomic extensions

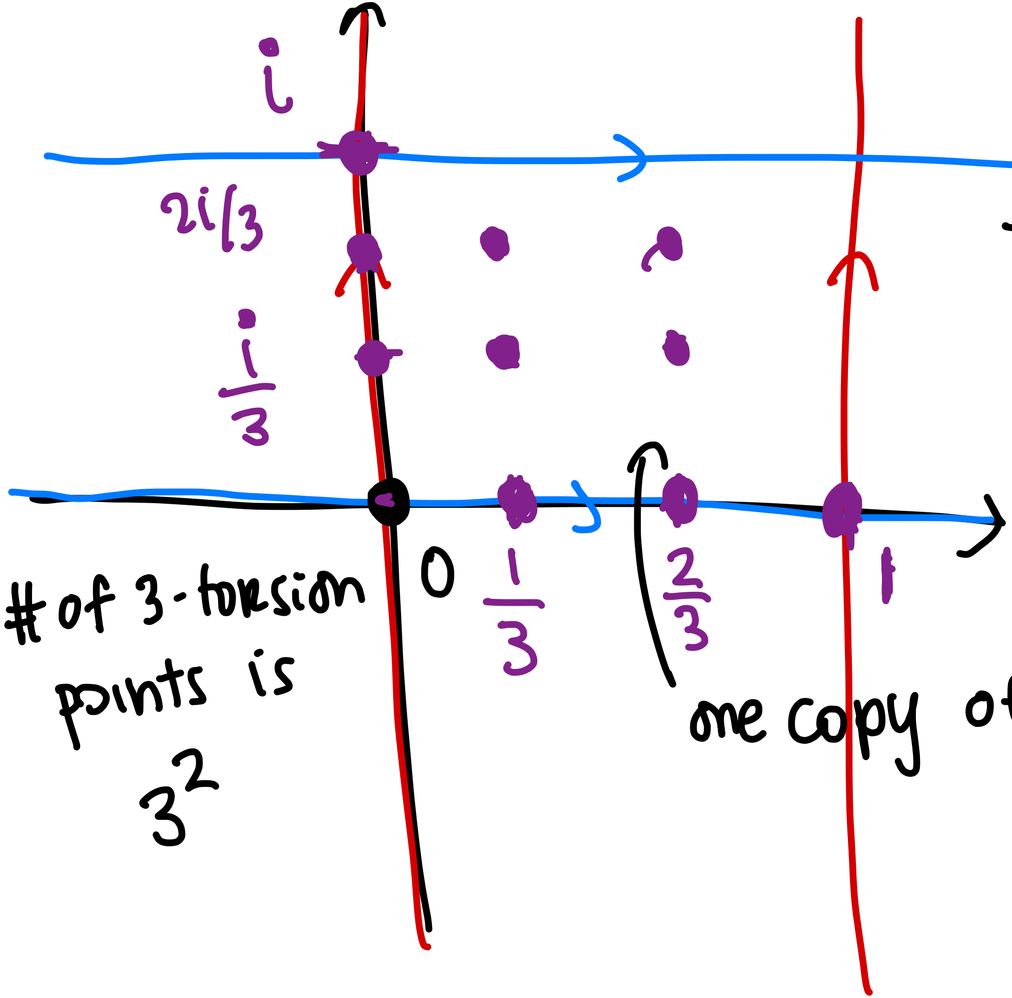
Next action that we know of is addition on elliptic curves



$\mathbb{Z} \curvearrowright \mathbb{C}$

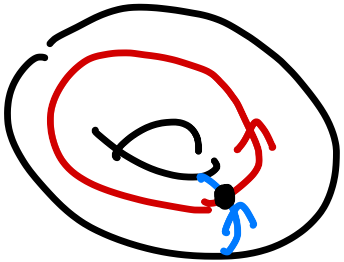
$\mathbb{Z} \curvearrowright \mathbb{C}/\Lambda$

ton of torsion



# of 3-torsion points is 3^2

one copy of the surface of the donut



$3 \cdot \frac{1}{3}$ in $\mathbb{C}$

$3 \cdot \frac{1}{3} = 0$ in the donut

Theorem $[K:\mathbb{Q}]=2$

Let $K=\mathbb{Q}(\sqrt{-d})$ $d>0$ an integer

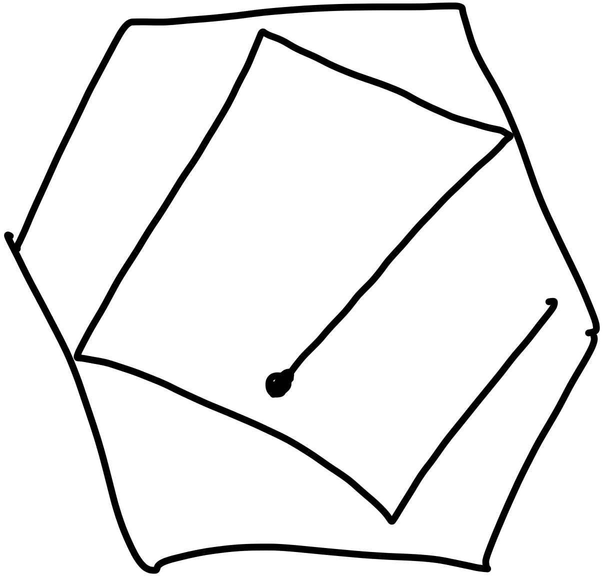
Let Λ be the lattice generated by 1 and either $\sqrt{-d}$ or $\frac{1}{2}+\frac{\sqrt{-d}}{2}$, depending on what d is,

then the torsion of the action of $\mathbb{Z}$ on $\mathbb{C}/\Lambda$ generates abelian extensions of $K=\mathbb{Q}(\sqrt{-d})$

If we also adjoin the j-invariant of $\mathbb{C}/\Lambda=E$, then we get all abelian extensions of K.

$[\mathbb{Q}:\mathbb{Q}]=1$ # of 3rd roots of unity = 3

billiards



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Donald in Mathemagic Land

That’s all for today!