Please turn in this assignment on Gradescope.

Problem 1 : (Objective C1)

a) Show that if $f(z) = \dfrac{az+b}{cz+d}$ is a fractional linear transformation, then $f^{-1}(z) = \dfrac{dz-b}{-cz+a}$.

   Hint: A function $g(z)$ is the inverse of a function $f(z)$ (in which case we denote it $f^{-1}(z)$) if and only if $f(g(z)) = z$ and $g(f(z)) = z$ for all $z \in \mathbb{C}$. In this case we say that f is *invertible*.

b) What is the domain of f?

c) What is the domain of f^{-1}?

d) Since f has a two-sided inverse, it is a bijection between two sets. What are those two sets?

Problem 2 : (Objective C3) What is the effect of each of these functions on the complex plane? Fill in the blanks for each function.

a) $f(z) = z + i$ is a translation in direction ‗‗‗ by amount ‗‗‗.

b) $f(z) = iz$ is a rotation around the point ‗‗‗ by angle ‗‗‗.

c) $f(z) = iz + 4$ is a rotation around the point ‗‗‗ by angle ‗‗‗.

d) $f(z) = (1+i)z$ is a rotation around the point ‗‗‗ by angle ‗‗‗, followed by scaling distances from the point ‗‗‗ by amount ‗‗‗.

e) $f(z) = 3z + 6i$ is scaling distances from the point ‗‗‗ by amount ‗‗‗.

f) $f(z) = 2 + 4i - z$ is a rotation around the point ‗‗‗ by angle ‗‗‗.

Hint: To answer these questions, ask yourself: Are there any points $z \in \mathbb{C}$ that are fixed by the function? How is the unit square (with vertices 0, 1, i, $1 + i$) transformed by the function?