Complex Analysis

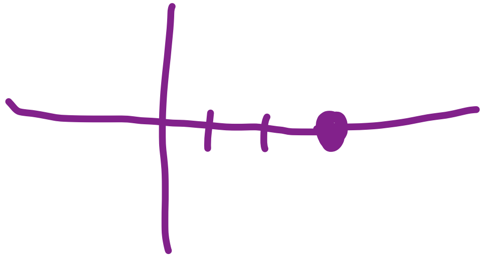
This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Redo HW 2 #2

$a_k = \left(\frac{1}{z-3}\right)^k$

a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{1}{z-3}\right)^k = 1 + (z-3)^{-1} + (z-3)^{-2} + \ldots$

centered at $z=3$



Ratio Test:

- Compute $\displaystyle\lim_{k\to\infty}\frac{|a_{k+1}|}{|a_k|} = L$

- the series converges if L exists and $L<1$

$$\lim_{k\to\infty} \left| \frac{1}{(z-3)^{k+1}} \right| \div \left| \frac{1}{(z-3)^{k}} \right|$$

$$|a_{k+1}| \div |a_k|$$

$$= \lim_{k\to\infty} \left| \frac{1}{(z-3)^{k+1}} \cdot \frac{(z-3)^k}{1} \right|$$

$$= \lim_{k\to\infty} \left| \frac{\cancel{(z-3)^k}}{(z-3)^{\cancel{k}+1}} \right|$$

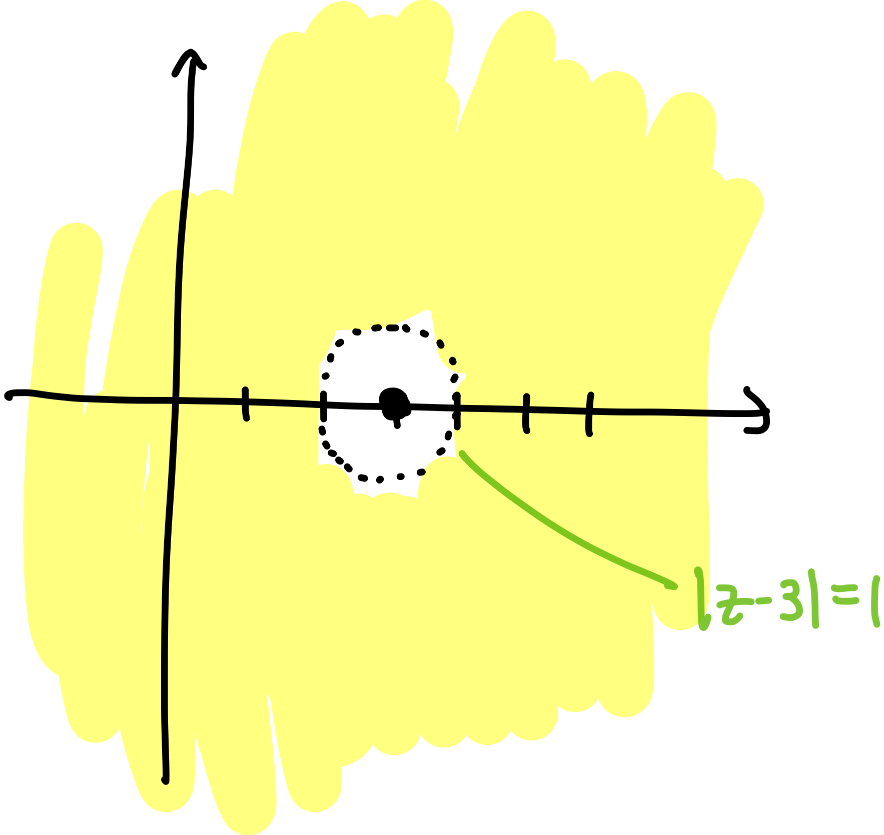
$$= \lim_{k\to\infty} \left| \frac{1}{z-3} \right| = \left| \frac{1}{z-3} \right| = \frac{1}{|z-3|} = L$$

exists if $z \neq 3$

For which values of z if

$$\frac{1}{|z-3|} < 1 \quad ?$$

$$\boxed{1 < |z-3|}$$

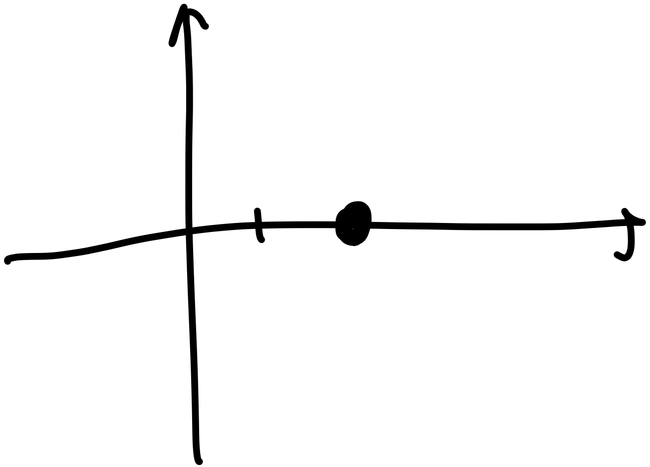


b) similar centered at $z=2$

$$\sum_{k=-2}^{\infty} \frac{(-1)^k (z-2)^k}{4^{k+3}} = \frac{(z-2)^{-2}}{4} - \frac{(z-2)^{-1}}{4^2} + \frac{1}{4^3} - \dots$$

$z=2$ will not be in region of convergence

$\rightarrow$ get an annulus



Happy to finish this on Friday!

D3: Solve an integral using Cauchy's Theorem:

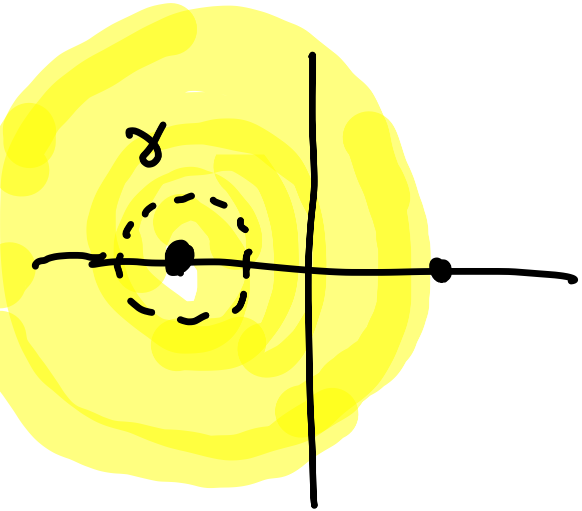
- Choose from these 5 <u>2</u> integrals that you can solve using Cauchy's Theorem (as one step)

- Do the integrals! Please clearly state the objective or technique you are using.

#1d) using the Laurent series

$$\int_\gamma \underbrace{\boxed{\frac{1}{(z-2)^2(z+2)}}}_{f(z)} dz = 2\pi i\, c_{-1}$$

where c_{-1} is the coefficient of the Laurent series of f centered at $z=-2$



We know f has such a Laurent series with region of convergence

$0 < |z+2| < 4$

$$f(z) = (z+2)^{-1} \cdot \underbrace{\frac{1}{(z-2)^2}}_{\sum_{k=0}^{\infty} b_k (z+2)^k}$$

$$= (z+2)^{-1}\left(b_0 + b_1(z+2) + b_2(z+2)^2 + \dots \right)$$

$$= b_0(z+2)^{-1} + b_1 + b_2(z+2) + \dots$$

$$= c_{-1}(z-2)^{-1} + c_0 + c_1(z+2) + \dots$$

So the c_{-1} of the Laurent series for f is the b_0 of the power series for $\frac{1}{(z-2)^2}$.

$$\frac{1}{(z-2)^2} = b_0 + b_1(z+2) + \ldots$$

$$\int_\gamma f(z)\,dz = 2\pi i \cdot \frac{1}{16} = \frac{\pi i}{8}$$

plug in $z=-2$

$$\frac{1}{(-2-2)^2} = b_0 + b_1 \cdot 0 + b_2 \cdot 0 + \ldots$$

$$\frac{1}{16} = b_0$$

If you wanted to compute

$$\frac{1}{(z-2)^2} = \sum_{k=0}^{\infty} b_k (z+2)^k$$

We did something similar on HW 8 #1b)

Use: $\frac{d}{dz}\left(\frac{1}{z-2}\right) = \frac{d}{dz}(z-2)^{-1} = -1(z-2)^{-2}\cdot 1$

$$= \frac{-1}{(z-2)^2}$$

Step 1: Find power series for $\frac{1}{z-2}$ centered at $z=-2$

Step 2: Differentiate & multiply by -1 to get the one you want

Recall that if $f(z) = \sum_{k=0}^{\infty} b_k (z - z_0)^k$

then $b_k = \dfrac{f^{(k)}(z_0)}{k!}$.

So $b_0 = f(z_0)$

$b_1 = f'(z_0)$

$b_2 = \dfrac{f''(z_0)}{2}$

That's all for today!