Complex Analysis

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Redo HW 1

#4 f'' is defined $f''(z) = 0 \quad \forall z \in U$

Let $g = f'$ then g is holomorphic on U

because $g' = f''$ which exists on all of U.

BMPS Theorem 2.17 (p.30)

$U \subseteq \mathbb{C}$ region $g: U \to \mathbb{C}$ cx-valued f^n

g' defined and $= 0 \quad \forall z \in U$ then g is constant.

So $\exists\, a \in \mathbb{C}$ such that $g(z) = a \quad \forall z \in \mathbb{C}$.

$\Rightarrow \quad f'(z) = a$

<u>Want</u> to apply Theorem 2.17 again

Note that we know that $f(z) = az + b$ in the end

<u>Hint</u>: Think of a function h with $h' = 0$ that will give you $f(z) = az + b$

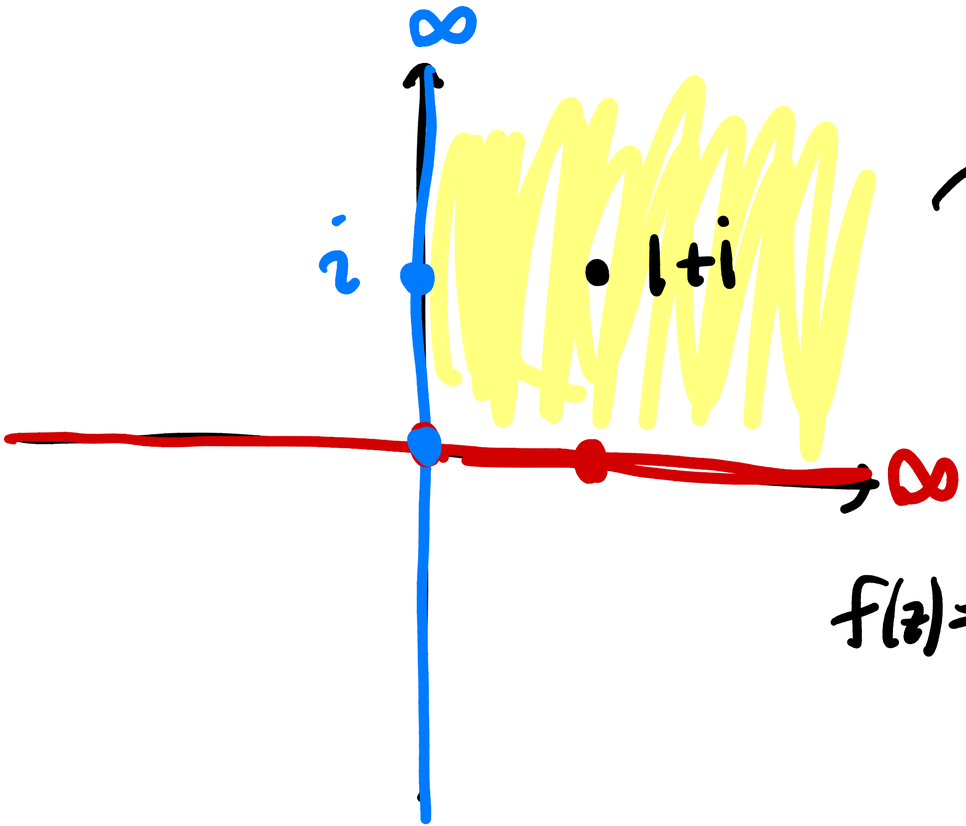
Think of some h s.t. $h' = 0$

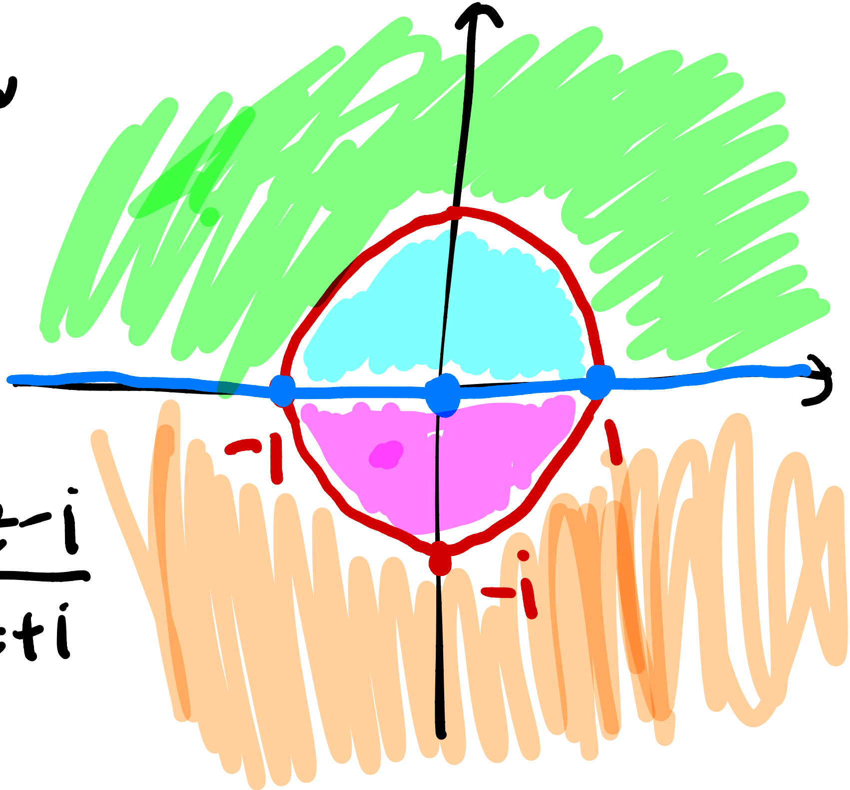
so $h(z) = b \in \mathbb{C}$

then solving for f in the formula

for h gives you $f(z) = az + b$.

6a)





$$f(z)=\frac{z-i}{z+i}$$

Image of the <u>real axis</u> line $0, 1, \infty$

$$f(0) = \frac{0-i}{0+i} = \frac{-i}{i} = -1$$

$$f(1) = \frac{1-i}{1+i}\,\frac{1-i}{1-i} = \frac{1-i-i-1}{1+1} = \frac{-2i}{2} = -i$$

$$f(\infty) = \lim_{z\to\infty} \frac{z-i}{z+i} = \lim_{z\to\infty} \frac{1-\frac{i}{z}}{1+\frac{i}{z}} = \frac{1-0}{1+0} = 1$$

Image of the imaginary axis $\quad 0, i, \infty$

$$f(0) = -1$$

$$f(\infty) = 1$$

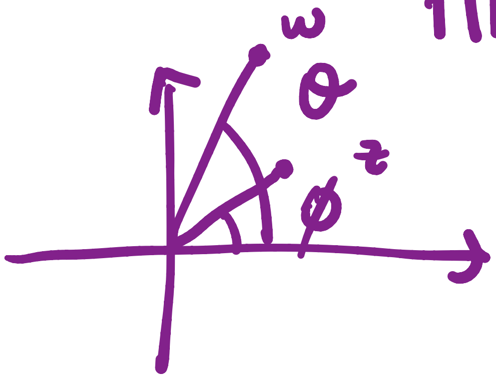
$$f(i) = \frac{i-i}{i+i} = \frac{0}{2i} = 0$$

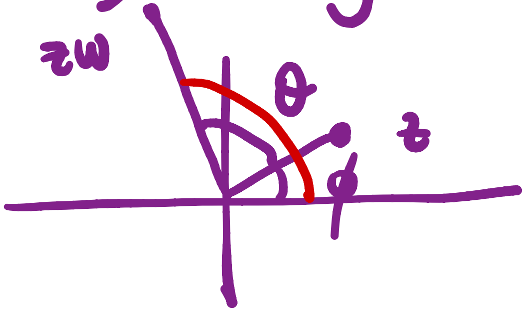
#8 a) is it true that $(e^z)^w = e^{zw}$? $\forall z, w \in \mathbb{C}$
$z, w \neq 0$

(this is <u>true</u> if $z, w \in \mathbb{R}$)

<u>No</u>

Big picture is that it is false that
$\mathrm{Arg}(zw) = \mathrm{Arg}(z) + \mathrm{Arg}(w)$





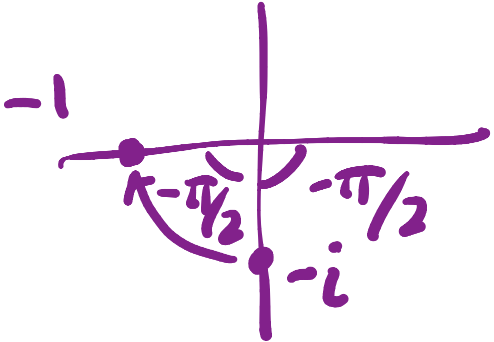
$\mathrm{Arg}(zw) = \theta + \phi$
$= \mathrm{Arg}\, z + \mathrm{Arg}\, w$

It is true that $\arg(zw) = \arg(z) + \arg(w)$

Counter example for Arg is

$z = -i \qquad w = -i$

$\mathrm{Arg}(z) = -\frac{\pi}{2} \qquad \mathrm{Arg}(w) = -\frac{\pi}{2}$ sum is $-\pi$



What is

$\mathrm{Arg}(-1) = \pi$

d) $(zw)^{1/2} = z^{1/2} \cdot w^{1/2}$

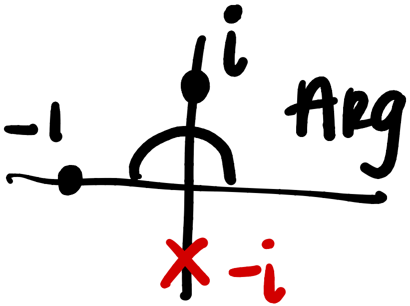
for real numbers

$\sqrt{zw} = \sqrt{z} \cdot \sqrt{w}$

$z = -1$

$w = -1$

$\sqrt{-1} = i$



$\text{Arg}(-1) = \pi$

$zw = 1 \quad \sqrt{1} = 1$

so $\quad ((-1)(-1))^{1/2} = 1 \neq -1 = i \cdot i = (-1)^{1/2} \cdot (-1)^{1/2}$

a) $z = x + iy$ $\quad$ $zw = (x+iy)(u+iv)$

$w = u + iv$

$$= xu + ixv + iuy - vy$$

$$= (xu - vy) + i(xv + uy)$$

$$e^{zw} = e^{xu-vy} \; e^{i(xv+uy)} = e^{xu-vy}\left(\cos(xv+uy) + i \sin(xv+uy)\right)$$

$$
\begin{aligned}
(e^z)^w &= \exp(w \,\mathrm{Log}(e^z)) \\
&= \exp(w \,\mathrm{Log}(e^x e^{iy})) \\
&= \exp(w(\ln|e^x| + i\,\mathrm{Arg}(e^{iy}))) \\
&= \exp(w \cdot (x + i\,\mathrm{Arg}(e^{iy}))) \\
&= \exp(wz)
\end{aligned}
$$

$z=i$

$w=i$

$zw=-1$

$\text{RHS} = e^{zw} = e^{-1}$

$\mathrm{Log}(e^{i}) = i$

$$
\begin{aligned}
(e^{i})^{i} &= \exp(i\,\mathrm{Log}(e^{i})) \\
&= \exp(i(\ln|1| + i\,1)) \\
&= \exp(i \cdot i) \\
&= \exp(-1)
\end{aligned}
$$

That's all for today!