

COMPLEX ANALYSIS

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

No exam in this class at all.

Instead, next week we'll have redo week 1

TOMORROW OR FRIDAY I will post Redo HW 1

Next week: Redo HW + Metacognition only

longer ~ 8-10 new problems

+ more repeat problems
from previous HWs

Idea:

Undergraduate credit: do as many OR as few problems as you want to improve your objective scores

Graduate credit: required to attempt all new problems, you may also do repeat problems to increase your scores

Redo week logistics

Monday, Wednesday, Friday will all be open
for questions

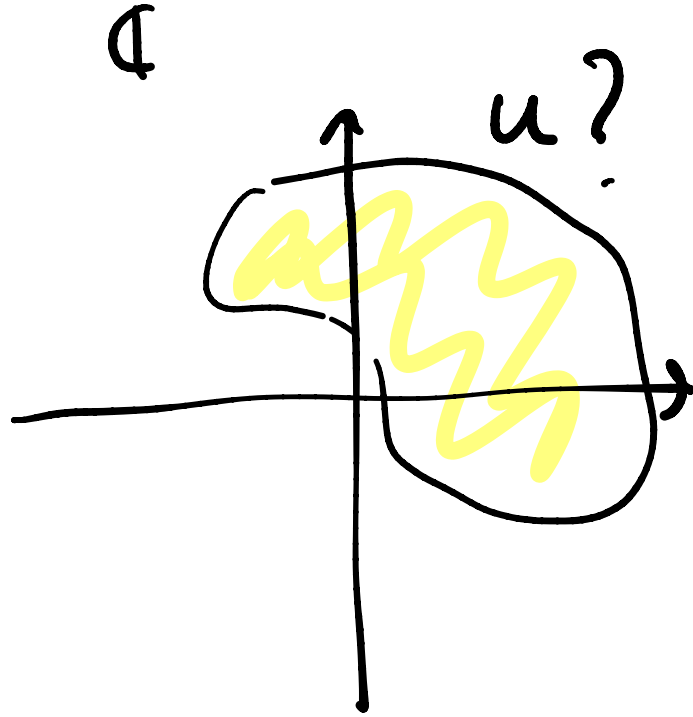
3 due dates: Monday, Wednesday, Friday 11:59pm

Have HW 3, 4 graded tonight

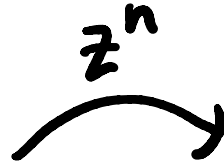
HW5 #1d)

$$U \subseteq \mathbb{C} \quad f(z) = z^n \quad n > 0 \quad n \in \mathbb{Z}$$

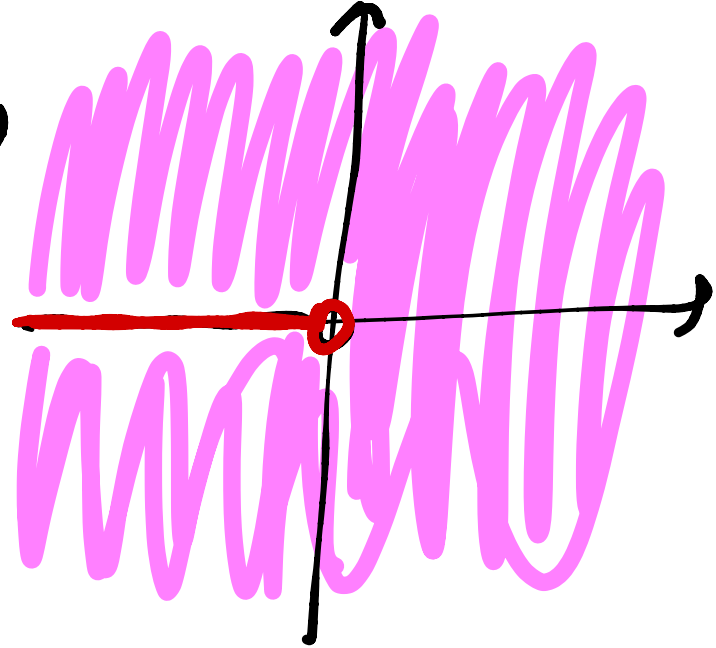
image covers $\mathbb{C} - (\{0\} \cup \{\text{neg real axis}\})$



z^n

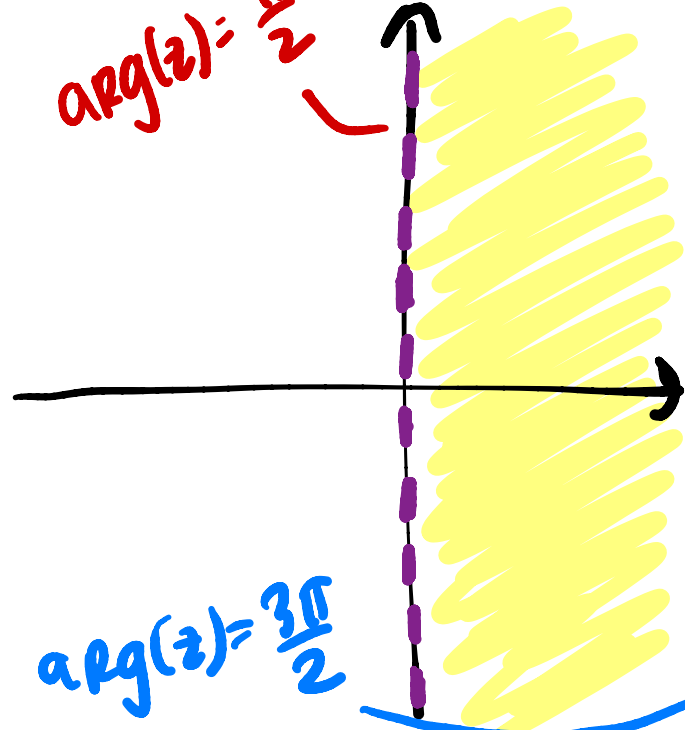


A curved arrow pointing from the yellow region U to the image region, with the label z^n above it.



First example $n=2$

$$\arg(z) = \frac{\pi}{2}$$

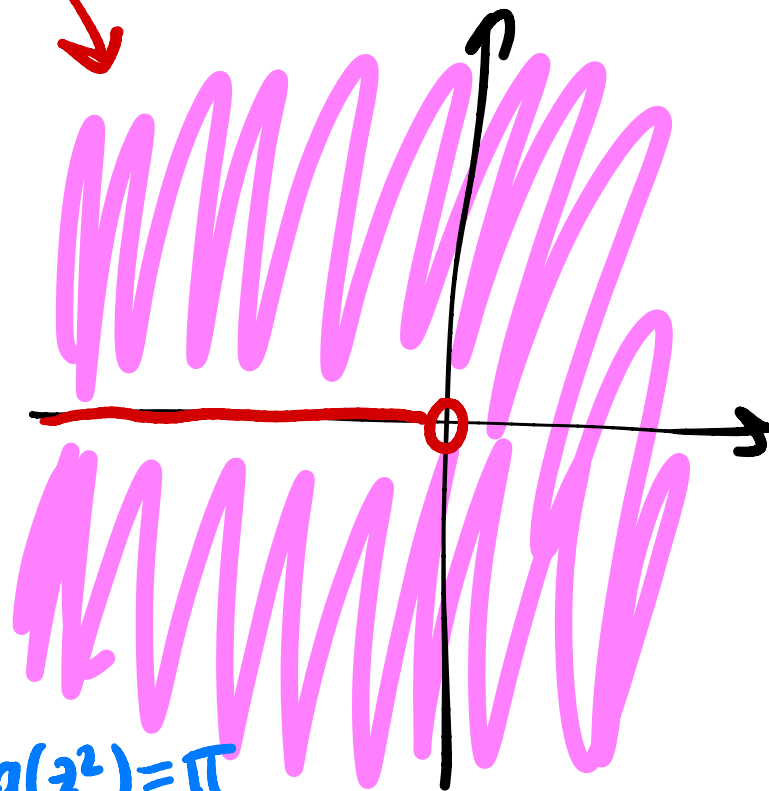


$$\arg(z) = \frac{3\pi}{2}$$

$$U = \{z \in \mathbb{C} : \operatorname{Re}(z) > 0\}$$

z^2

$$\arg(z^2) = \pi$$



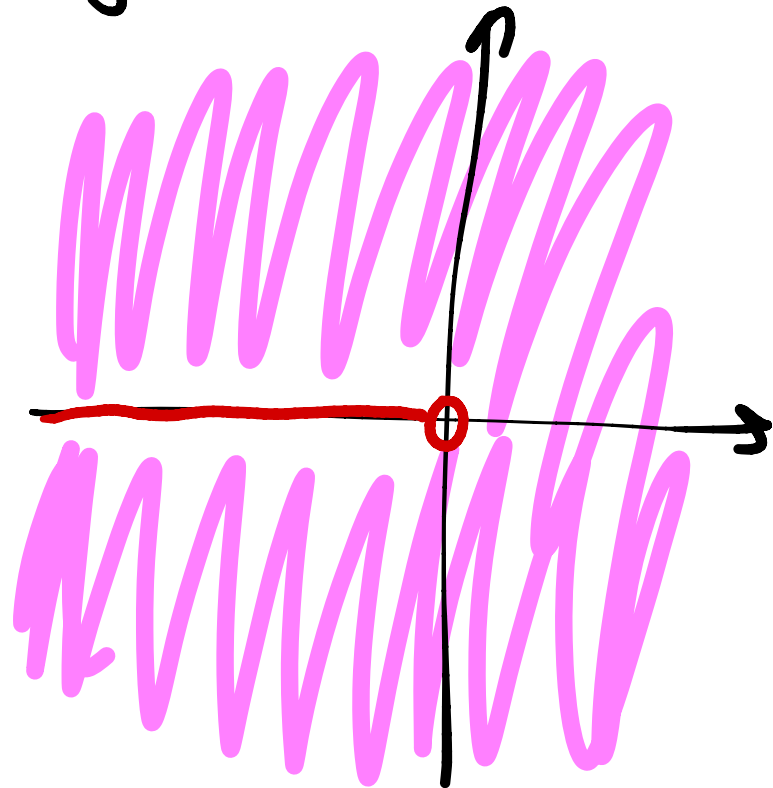
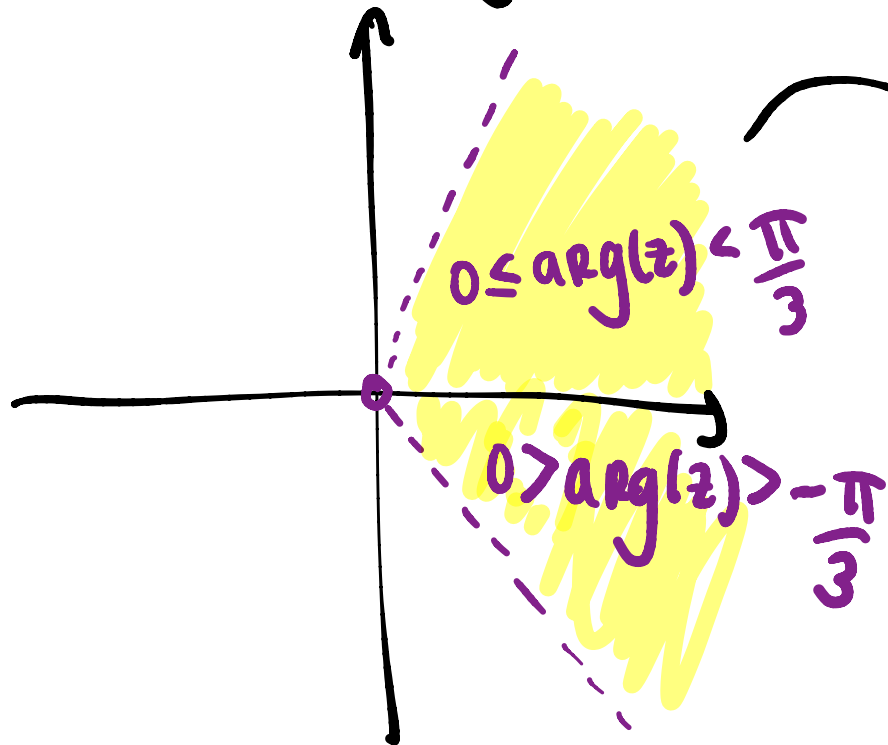
$$\arg(z^2) = \pi = 3\pi$$

Next $n=3$

$$\arg(z) = \phi$$

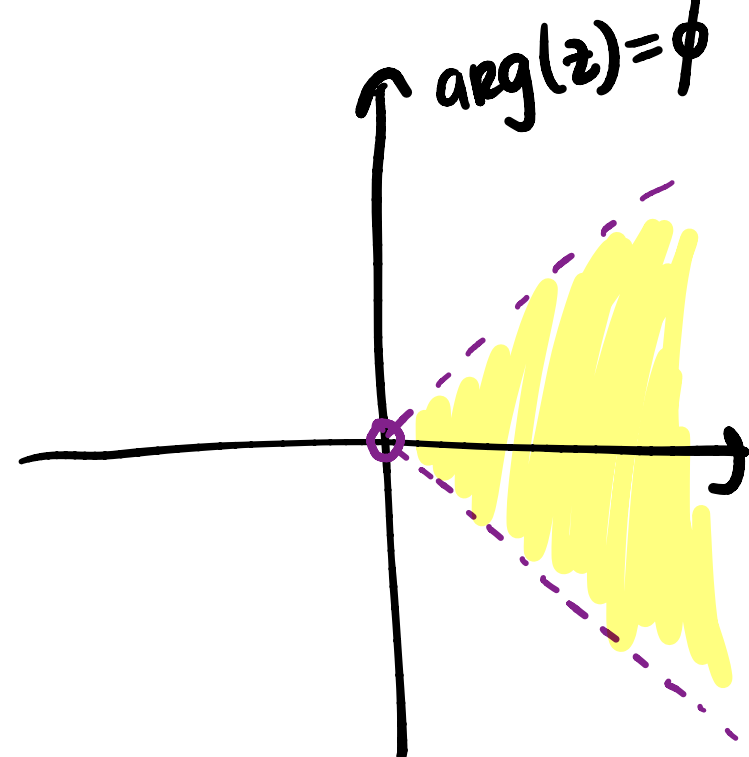
$$z^3$$

$$\arg(z^3) = 3\phi$$

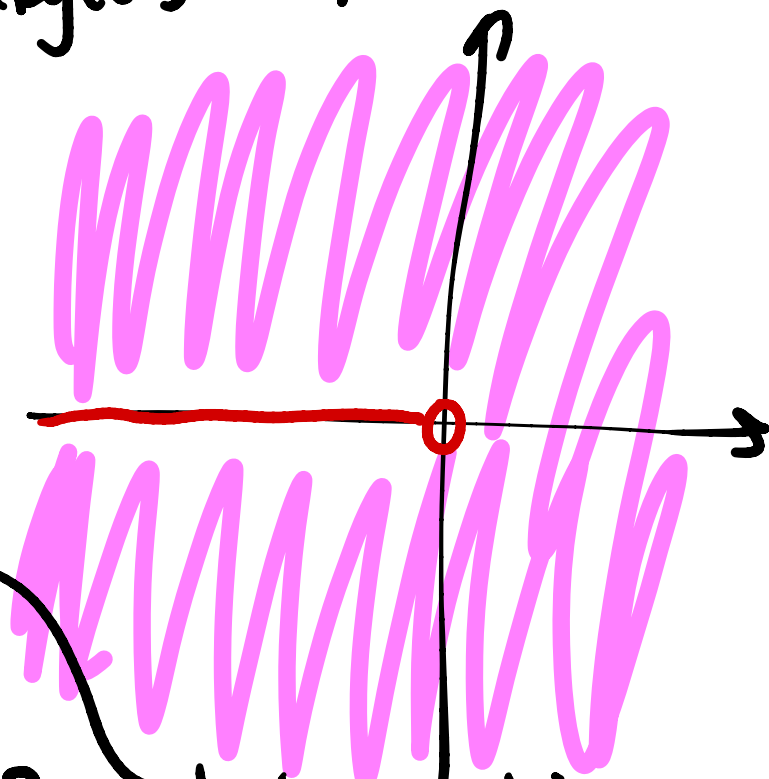


$$U = \left\{ z \in \mathbb{C} : -\frac{\pi}{3} < \arg(z) < \frac{\pi}{3} \right\}$$

General $n > 0, n \in \mathbb{Z}$



z^n $\arg(z^n) = n \cdot \phi$



$\sqrt[n]{z}$



$$U = \left\{ z \in \mathbb{C} : -\frac{\pi}{n} < \arg(z) < \frac{\pi}{n} \right\}$$

holomorphic on its domain

$$U = \left\{ z \in \mathbb{C} : -\frac{\pi}{n} < \arg(z) < \frac{\pi}{n} \right\}$$

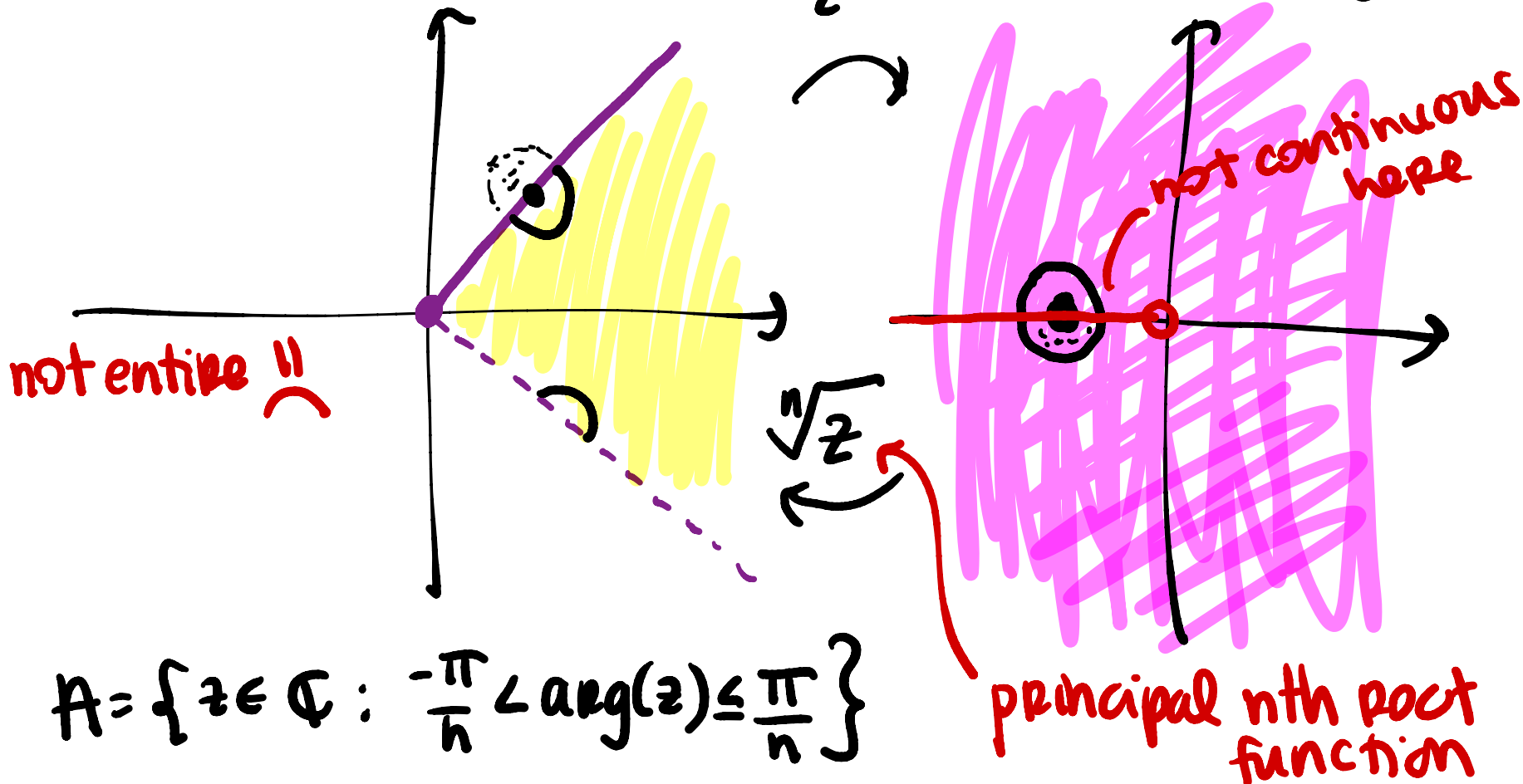
If you include $\arg(z) = \frac{\pi}{n}$



$\arg(z^n) = \pi$ → neg real axis

What if?

surjective yay!



HW5 #5 → Friday

Notation

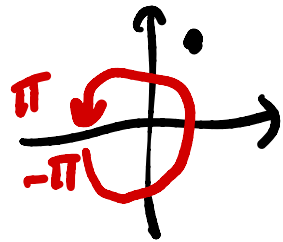
$$\text{Log}(z) = \ln|z| + i \text{Arg}(z)$$

$$\text{log}(z) = \underline{\ln|z|} + i \text{arg}(z)$$

} branch of the
"inverse" of
 $\exp(z)$

In real function that is the
inverse of e^x

$\text{Arg}(z)$ = principal branch of $\text{arg}(z)$
outputs in the set $(-\pi, \pi]$



$\text{arg}(z) = \{ \text{all angles associated to } z \}$

↑ multivalued

$\sqrt{4} = 2$ "principal branch"
= square root(4)

or = square root(4)
 $\sqrt{4} = \{2, -2\}$ multivalued function

THAT'S ALL FOR TODAY!

Today OH 4pm-5pm