Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Check in

Any questions or concerns? Anything unclear?

Questions on objective A8

A8: Plot complex numbers in the plane (rectangular and polar forms), and the sum and product of complex numbers in the plane

Questions on objective A8

It is covered by Problem 1 of HW 2 (plot sums, differences, and products of numbers in polar or rectangular forms).

Questions on objective A9

A9: Sketch sets and parametric curves in the complex plane; recognize simple closed contours.

path

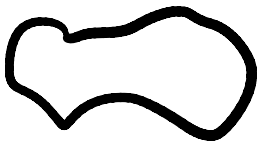
curve

contour

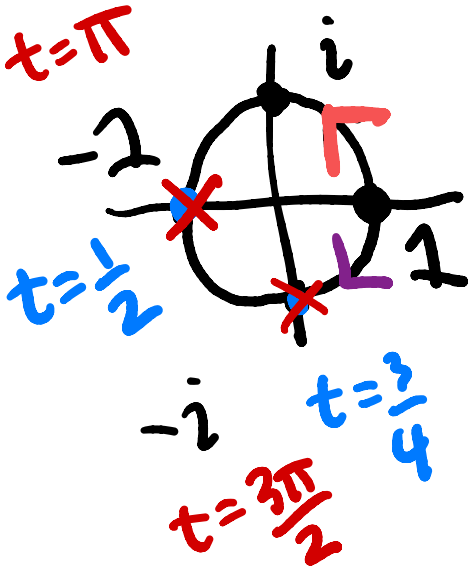
A 1st course in cx analysis: basically all the same

Complex variables

$\gamma : [a,b] \to \mathbb{C}$ path



curve/contour is the image or picture of γ in $\mathbb{C}$



circle of radius 1 centered at 0

a contour or curve

Can be parametrized / made into a path in many ways

① $\gamma(t) = e^{it}$ $\quad 0 \le t \le 2\pi$

② $\gamma(t) = e^{2\pi i t}$ $\quad 0 \le t \le 1$

③ clockwise

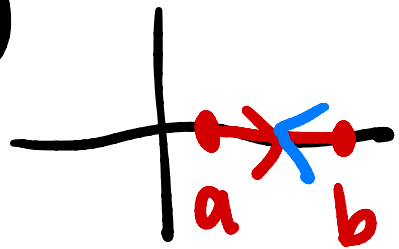
$\gamma(t) = e^{-it}$ $\quad 0 \le t \le 2\pi$

$$\int_\gamma f(z)\,dz$$ counterclockwise

$$= -\int_{-\gamma} f(z)\,dz$$

← same contour as γ but clockwise

($\int_a^b f(x)\,dx = F(b)-F(a)$ $= -\int_b^a f(x)\,dx = F(a)-F(b)$)



What matters for integrals is

- contour (shape in plane)
- orientation

Not: parametrization

Questions on objective A9

It is covered by Problem 2 of HW 2 (sketch sets and contours in the complex plane).

Lastly, objective A10

easy 1st step plot points ← A9: Given γ draw picture

A10: Given a picture, give γ.

A10: parametrize contours

Most likely contours we will parametrize:

- circles of all radii and centers,
- lines and rectangles,
- graphs of simple functions.

$y=x^2$ $y=x^3$ ...

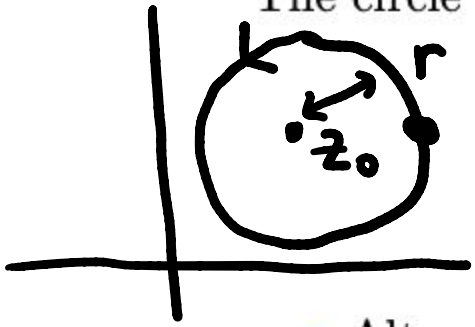
$y=\sin x$ $y=e^x$

in math: line is always a straight line

line

not a line

Circles of all centers and radii



counterclockwise

The circle with radius r and center z_0 is parametrized by

$$\gamma(t) = z_0 + re^{it}, \quad 0 \le t \le 2\pi.$$

radius

$\gamma(t) = z_0 + re^{\pi it}$

$0 \le t \le 2$

- Alternatively: $\gamma(t) = z_0 + re^{2\pi it}, \quad 0 \le t \le 1.$
- What about only part of the circle? (top half, left half, etc.)

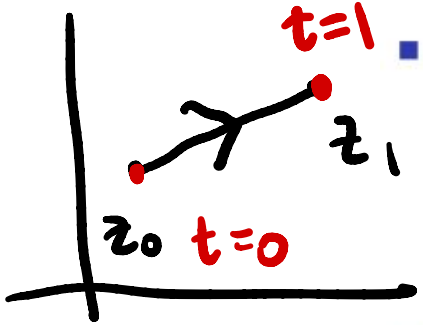
$0 \le t \le \pi$

$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

- What about going clockwise?

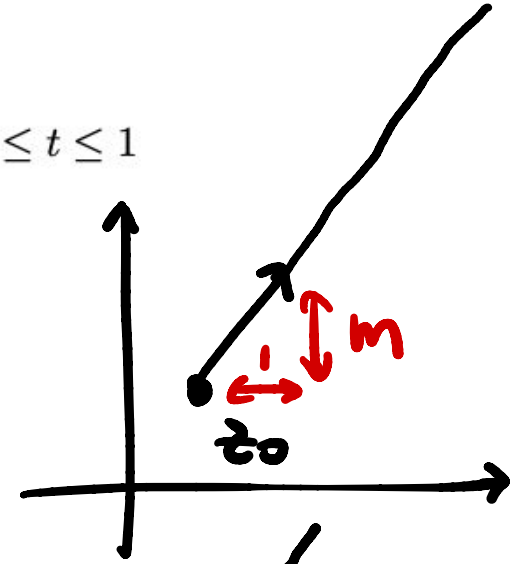
↳ do e^{-it} instead

Lines and rectangles



- Line from z_0 to z_1:

$$\gamma(t) = z_0(1-t) + z_1 t = z_0 + t(z_1 - z_0), \quad 0 \le t \le 1$$

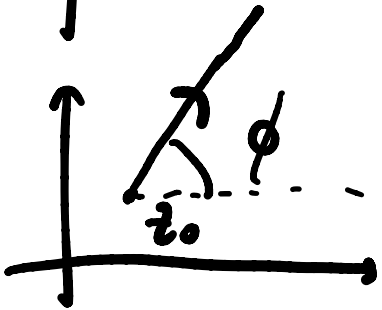


- Line starting at z_0 of slope m:

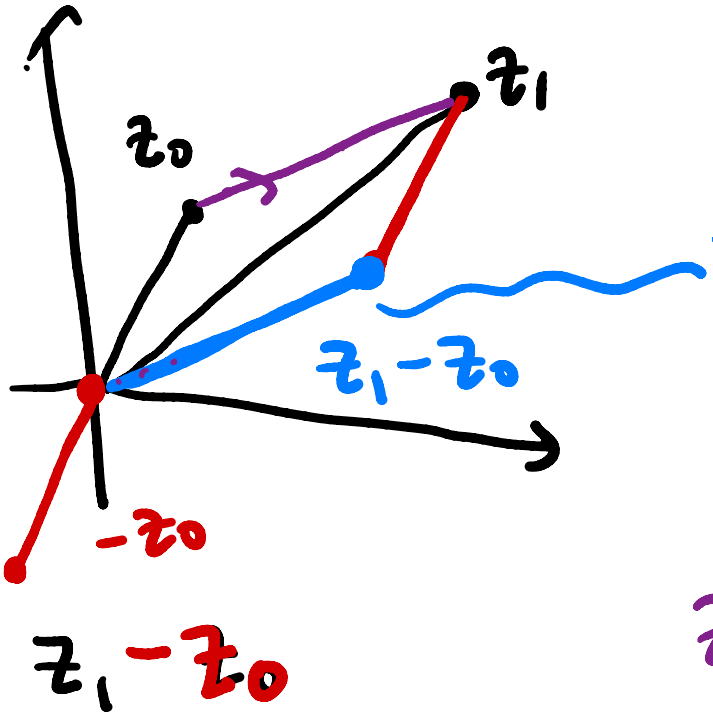
$$\gamma(t) = z_0 + t(1 + im), \quad 0 \le t$$

- Ray emanating from z_0 at angle ϕ from real axis:

$$\gamma(t) = z_0 + te^{i\phi}, \quad 0 \le t.$$



Lines and rectangles



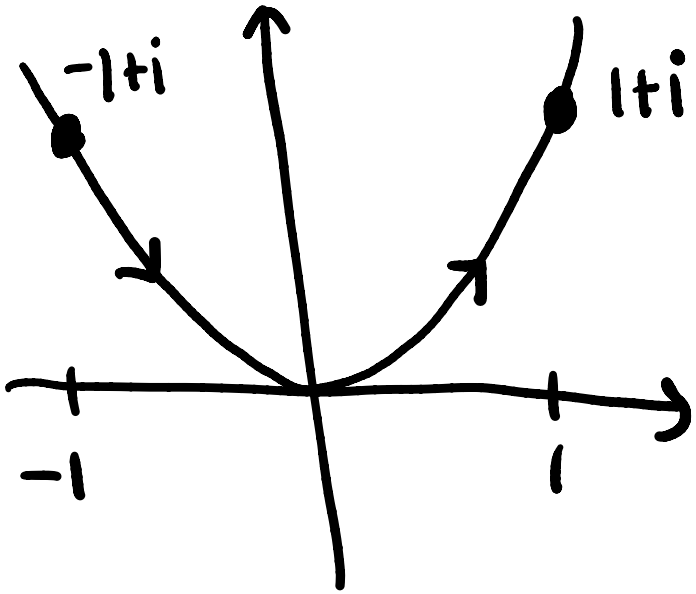
to parametrize this

$t(z_1 - z_0)$ $\quad 0 \le t \le 1$

$z_0 + t(z_1 - z_0)$ $\quad 0 \le t \le 1$

Graphs of simple functions

Parametrize the contour given by the curve $y = x^2$, $-1 \leq x \leq 1$.



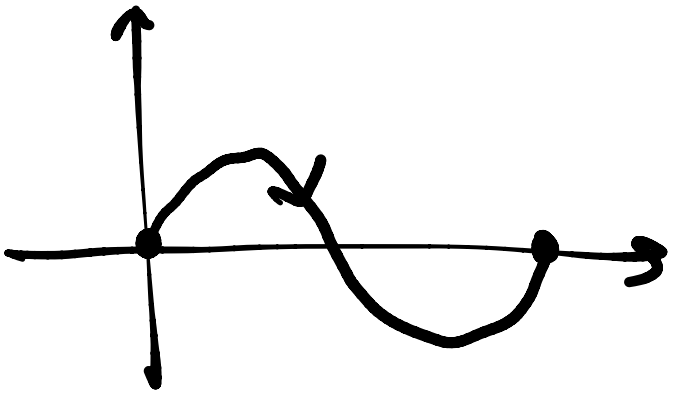
Let $t = x$

$\gamma(t) =$ Real part $+ i$ Im part

$x(t)$ $\quad$ $y(t)$

$\gamma(t) = t + i t^2 \qquad -1 \leq t \leq 1$

$y = \sin x \quad 0 \leq x \leq 2\pi$ Let $x = t$



$\gamma(t) = t + i\sin t$

$0 \leq t \leq 2\pi$

That's all for today!