

COMPLEX ANALYSIS

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

ANNOUNCEMENTS

- Course policies and syllabus are up on the course website!
- Please continue to turn in all work on Gradescope, in one high-quality pdf.
- Warm ups are not graded (only on completion).

#SCHOLARSTRIKE

From scholarstrike.com:

Scholar Strike is both an action, and a teach-in. Some of us will, for two days (September 8-9), refrain from our many duties and participate in actions designed to raise awareness of and prompt action against racism, policing, mass incarceration and other symptoms of racism's toll in America.

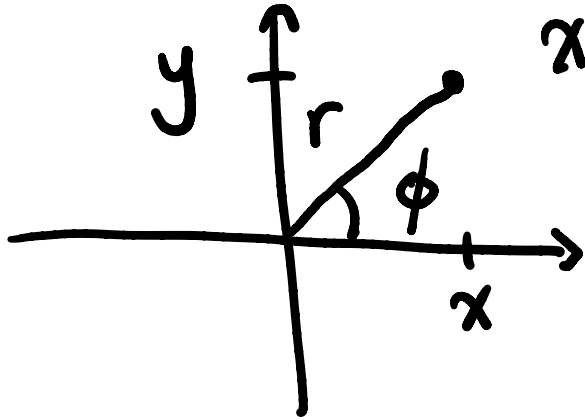
Please feel free to participate in whatever way you see fit.

CHECK IN

Any questions or concerns? Anything unclear?

WARM UP 1.2

#2



$$x+iy = re^{i\phi}$$

r = modulus OR
absolute value

ϕ = argument

x = Real part

y = imaginary part

$$a) \frac{z-a}{z+a}$$

$$a \in \mathbb{R}$$

$$z = u + iv$$

multiply top &
bottom by conj
of denom
↓

$$\left. \begin{array}{l} z+a \\ = \\ x+iy \end{array} \right\}$$

$$\frac{(u+iv-a)}{\boxed{(u+iv+a)}} = \frac{[(u-a)+iv] \cdot [(u+a)-iv]}{[(u+a)+iv] [(u+a)-iv]}$$

conjugate

$$(u+a)+iv \rightsquigarrow (u+a)-iv$$

$$\frac{[(u-a)+iv] \cdot [(u+a)-iv]}{[(u+a)+iv] [(u+a)-iv]} = \frac{(u-a)(u+a) - iv(u-a) + iv(u+a) + v^2}{(u+a)(u+a) + \cancel{(u+a)(-iv)} + iv\cancel{(u+a)} + v^2}$$

$$= \frac{[u^2 - a^2 + v^2] + i[vu + va - vu + va]}{u^2 + 2au + a^2 + v^2}$$

$$\frac{[u^2 - a^2 + v^2] + i[v\cancel{u} + va - v\cancel{u} + va]}{u^2 + 2au + a^2 + v^2}$$

$$u^2 + 2au + a^2 + v^2$$

$$= \frac{u^2 - a^2 + v^2}{u^2 + 2au + a^2 + v^2} + i \frac{2va}{u^2 + 2au + a^2 + v^2}$$

$x + i$
Real part

y
imaginary part

b) i^n

$$= 0 + 1 \cdot i$$

$$i^1 = i$$

$$\operatorname{Re}(i) = 0 \quad \operatorname{Im}(i) = 1$$

$$i^2 = -1$$

$$\operatorname{Re}(i^2) = -1 \quad \operatorname{Im}(i^2) = 0$$

$$i^3 = -i$$

$$\operatorname{Re}(i^3) = 0 \quad \operatorname{Im}(i^3) = -1$$

$$i^4 = 1$$

$$\operatorname{Re}(i^4) = 1 \quad \operatorname{Im}(i^4) = 0$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$

$$i^8 = 1$$

$$i^{4k+1} = i$$

$$i^{4k+2} = -1$$

$$i^{4k+3} = -i$$

$$i^{4k} = 1$$

$$i^n = \begin{cases} i & \text{if } n \equiv 1 \pmod{4} \\ -1 & \text{if } n \equiv 2 \pmod{4} \\ -i & \text{if } n \equiv 3 \pmod{4} \\ 1 & \text{if } n \equiv 0 \pmod{4} \end{cases}$$

#2

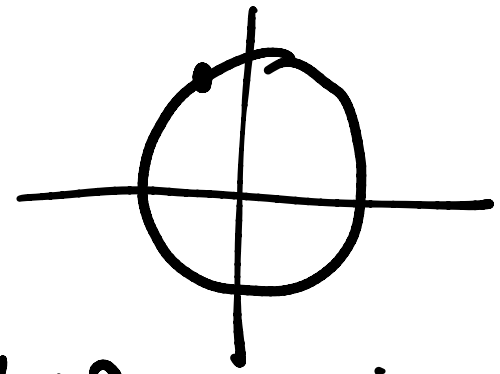
$$c) \left(\frac{-1+i\sqrt{3}}{2} \right)^3 = \dots = 1$$

$$\begin{aligned} \operatorname{Re}(\dots) &= 1 \\ \operatorname{Im}(\dots) &= 0 \end{aligned}$$

$$-\frac{1}{2} = \cos \frac{2\pi}{3}$$

$$\frac{\sqrt{3}}{2} = \sin \frac{2\pi}{3}$$

$$\frac{-1+i\sqrt{3}}{2} = e^{i\frac{2\pi}{3}}$$



$$\left(e^{i\frac{2\pi}{3}} \right)^3 = e^{2\pi i} = 1$$

#3 $|z|=1$ if and only if $\frac{1}{z} = \bar{z}$

$$|z|=1 \Rightarrow \frac{1}{z} = \bar{z} \quad \& \quad \frac{1}{z} = \bar{z} \Rightarrow |z|=1$$

Let $z = x + iy$. If $|z|=1$, then $\sqrt{x^2 + y^2} = 1$

OR $x^2 + y^2 = 1$. Then

$$\frac{1}{z} = \frac{1}{x+iy} \cdot \frac{(x-iy)}{(x-iy)} = \frac{x-iy}{x^2 - ixy + ixy + y^2} = \frac{x-iy}{x^2 + y^2}$$

$$\frac{1}{z} = \dots = \frac{x-iy}{x^2+y^2} = \frac{x-iy}{1} = x-iy = \bar{z}$$

so $\frac{1}{z} = \bar{z}.$

When we write $e^{i\phi}$ what can ϕ be?

Answer 1: anything!

$$1 = e^{2\pi i}$$

$$\phi = 0$$

$$\phi = 2\pi \quad \checkmark$$

$$1 = e^{4\pi i}$$

$$\phi = 4\pi \quad \checkmark$$

use notation
 $\phi = \arg z$



← $\phi = \text{Arg } z$

correct!

Answer 2: Agree in advance that ϕ takes values in some interval

commonly: $(-\pi, \pi]$ or $[0, 2\pi)$

LIGHTNING ROUND 1

What is the real part of $\frac{1+i}{1-i}$?

LIGHTNING ROUND 1 ANSWER

What is the real part of $\frac{1+i}{1-i}$?

- a. 0
- b. 1
- c. i
- d. Don't know

$$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+2i-1}{1^2+1^2} = \frac{2i}{2} = i$$

$$\text{so } \operatorname{Re}\left(\frac{1+i}{1-i}\right) = 0$$

LIGHTNING ROUND 2

What is $|3 - 4i|$?

LIGHTNING ROUND 2 ANSWER

What is $|3 - 4i|$?

- a. $\sqrt{-7}$
- b. 5
- c. 25
- d. Don't know

$$|3-4i| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

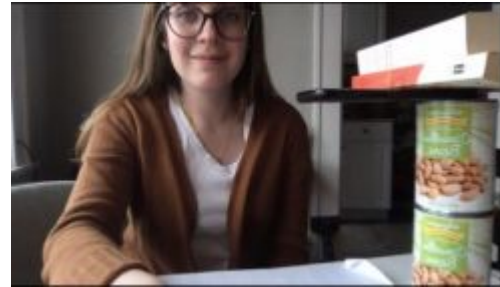
PEER INSTRUCTION: HOW TO COMMUNICATE MATH

Unfortunately Teams do not support LaTeX/MathJax :(

If someone has a tablet with Teams installed, they can join the meeting with their tablet and share the screen.

During a video meeting, in the Share options, you can choose Whiteboard to have a canvas where everyone can write.

You can get more creative!



(photo from <https://www.rochester.edu/college/cetl/assets/pdf/diy-document-camera.pdf>)

PEER INSTRUCTION: TECHNICAL DETAILS

From now on, I will assign one person to begin the video call; everyone else please wait then join.

You can record the video meeting but please obtain everyone's consent first.

Please note that everyone can read the chat by navigating to the channel, and everyone will be able to view the video.

PEER INSTRUCTION PROBLEM 1

Find all solutions to the following equations:

a. $z^6 = 1$

b. $z^6 = -9$

PEER INSTRUCTION PROBLEM 2

Show that z is a real number if and only if $z = \bar{z}$.

PEER INSTRUCTION PROBLEM 3

Suppose that p is a polynomial with real coefficients and z is a complex number. Prove that $\overline{p(z)} = p(\bar{z})$.

THAT'S ALL FOR TODAY!