Math 295 - Fall 2020
Homework 9
Due at 11:59pm on Friday November 13

Please turn in this assignment on Gradescope.

Problem 1 : (Objectives E4, E5) Consider the series

$$f(z) = \sum_{k=0}^{\infty} \left(\frac{1}{z-3} \right)^k.$$

a) What is the region of convergence of this series?

b) What function f is represented by this series?

c) Is this a Laurent series for the function f which is your answer to part b)? If so, what is the value z_0 such that this Laurent series is centered at z_0?

Problem 2 : (Objective E5) Find the three Laurent series of

$$f(z) = \frac{3}{(1-z)(z+2)}$$

centered at 0, which are defined on the regions $|z| < 1$, $1 < |z| < 2$, and $2 < |z|$, respectively.

Problem 3 : (Objectives E4, E5, E6)

a) Find a Laurent series for

$$\frac{1}{(z^2-4)(z-2)}$$

centered at $z = 2$, and give the region in which it converges.

b) Compute the integral

$$\int_\gamma \frac{1}{(z^2-4)(z-2)}\, dz,$$

where γ is the circle of radius 1 centered at 2.

Problem 4 : (Objective E6) Recall from Homework 8 that the power series of $\exp(z)$ centered at $z = -1$ is

$$e^{-1} \sum_{k=0}^{\infty} \frac{(z+1)^k}{k!}.$$

Compute the integral

$$\int_\gamma \frac{\exp(z)}{(z+1)^{34}}\, dz,$$

where γ is the circle of radius 2 centered at -2.