Math 295 - Fall 2020
Homework 8
Due at 11:59pm on Friday November 6

Please turn in this assignment on Gradescope.

This is the final version of the homework.

Problem 1 : (Objectives E1, E2) For each of the following complex functions, find a power series centered at $z_0 = 0$, and give the radius of convergence of the series.

a) $\dfrac{1}{1+4z}$

b) $\dfrac{z^2}{(4-z)^2}$

c) $\cos(z^2)$

Problem 2 : (Objectives E1, E2) In this problem, consider the complex function given by the rule $f(z) = \dfrac{1}{z^2+1}$.

a) Give a power series centered at $z_0 = 0$ for this function.

b) What is the radius of convergence of this power series?

c) What is the region $U \subset \mathbb{C}$ on which f is holomorphic?

d) Read carefully Corollary 8.9 in BMPS. What is the distance between $z_0 = 0$ and the boundary of U? (This is the closest the "edge" of U comes to the point $z_0 = 0$.) Does your answer support Corollary 8.9?

e) Consider now the *real* function $g(x) = \frac{1}{x^2+1}$. Where is this function differentiable on the real line? What is the distance between $x_0 = 0$ and the boundary of the set where the real function g is differentiable? Does this contradict Corollary 8.9?

Problem 3 : (Objective E3) Prove that $f(z) = \exp(z)$ is analytic at every point $z_0 \in \mathbb{C}$ by giving a power series expansion for f around an arbitrary point $z_0 \in \mathbb{C}$.